B8 · Energy conservation and 2-D motion of charges

Completed notes

Use energy conservation, then split a sideways-entering charge into constant-speed and accelerated motion.

Slide 1 of 7 · Energy conservation for a charge in a field

Ep means electric potential energy and Ek means kinetic energy, as in B6. “At A” and “at B” name the two points. Only the electric force does work, so total energy **stays the same**: Ep(A) + Ek(A) = **Ep(B) + Ek(B)**.

If the charge starts **at rest**, Ek(A) = 0, so Ek(B) = **Ep(A) − Ep(B)**. The potential energy lost becomes **kinetic energy**.

Then ½mv² = Ek, so v = **√(2Ek/m)**. Use the object’s own **mass** (the ball’s, not an electron’s).

The charge q is not needed when the energies are given. To compare speeds at two points, compare the **kinetic energies**: more kinetic energy means more speed for the same mass.

Example. A charged pith ball ($m = 2.00 \times 10^{-4}$ kg, $q = 1.5 \times 10^{-8}$ C) is held at A in a uniform field with 4.0×10^{-7} J of electric potential energy, then released. At B it has 2.5×10^{-8} J of electric potential energy. Find its speed at B.

1. It starts from rest, so Ek(A) = 0 and Ek(B) = $4.0 \times 10^{-7} - 2.5 \times 10^{-8} = 3.75 \times 10^{-7}$ J.

2. $\frac{1}{2}mv^2 = 3.75 \times 10^{-7}$ J, so $v = \sqrt{(2 \times 3.75 \times 10^{-7} \div (2.00 \times 10^{-4}))} = 6.12 \times 10^{-2}$ m/s.

3. The charge was not needed.

Try it. The same pith ball is released from rest at A, which has 4.0×10^{-7} J of electric potential energy. It reaches a point C where its electric potential energy is 1.0×10^{-7} J. Find its kinetic energy at C and say whether its speed there is greater or less than at B (where Ek = 3.75×10^{-7} J).

1. Total at A equals total at C, and it starts at rest: Ek(C) = Ep(A) − Ep(C) = 4.0×10^{-7} J − 1.0×10^{-7} J = 3.0×10^{-7} J.

2. At B the kinetic energy was 3.75×10^{-7} J, which is greater, so the ball is moving faster at B than at C.

Slide 2 of 7 · A charge entering a uniform field sideways

A charge entering a field sideways is like a ball rolling off a table: it keeps moving forward while a **force** pulls it down. Here the force is **electric**, and gravity is too small to matter. The horizontal speed v_x is the speed **along the plates**.

The electric force is at **right angles** to the plates and points toward the plate of the opposite sign (an electron toward the **positive** plate). Motion does not need a force in its own direction.

The two directions do not **affect** each other. Horizontal: net force **zero**, acceleration zero, constant velocity v_x, **uniform** motion. Vertical: constant force, constant acceleration $a = \mathbf{qE/m}$, speed grows from zero, **uniformly accelerated** motion.

Together these make a **parabola**. In explanations say “net force zero” or “constant force” in each direction.

Example. An electron enters the uniform field between parallel plates moving horizontally with speed v_x. Gravity is too small to matter. The top plate is negative and the bottom plate is positive. Complete the table for horizontal and vertical motion (net force, acceleration, velocity, motion type), then name the shape of the path.

1. Horizontal: net force zero, acceleration zero, constant velocity v_x, uniform motion.

2. Vertical: constant electric force toward the positive plate, constant acceleration, vertical speed grows from zero, uniformly accelerated motion.

3. The path is a parabola.

Try it. A proton enters the same field at the same horizontal speed as the electron, beside the top plate (negative top, positive bottom). Compare the proton’s path with the electron’s path in direction and in how sharply it bends.

1. The proton is positive, so the force on it points toward the negative plate. The electron was pushed toward the positive plate, so the two curves bend in opposite directions.

2. The charges are equal in size, so the forces are equal. The proton’s mass is much larger, so $a = qE/m$ is much smaller and its path is flatter. Both paths are parabolas.

Slide 3 of 7 · From acceleration back to voltage

To go from motion back to voltage, work the chain backward from what you want, then forward with numbers. The electric force is the only force, so F = **ma**.

Then E = **F/q**, where q is the charge that feels the force (use the electron's size, 1.60×10^{-19} C; the direction is found separately), and then $\Delta V = \mathbf{E\Delta d}$ (multiply both sides of $E = \Delta V/\Delta d$ by Δd). Each formula has three quantities, so each step is one **small calculation**, with units.

Write the chain of formulas **first**, before any numbers: a → F → E → ΔV. Use the electron's mass and charge from the data sheet.

Never use $\Delta V = \Delta Ep/q$ here: the **energy** is not given.

Example. An electron ($m = 9.11 \times 10^{-31}$ kg, $q = 1.60 \times 10^{-19}$ C) between parallel plates has an acceleration of 3.00×10^{15} m/s². The plates are 0.0360 m apart. Find the potential difference between the plates.

1. $F = ma = (9.11 \times 10^{-31})(3.00 \times 10^{15}) = 2.733 \times 10^{-15}$ N.

2. $E = F/q = 2.733 \times 10^{-15} \div (1.60 \times 10^{-19}) = 1.708 \times 10^{4}$ N/C.

3. $\Delta V = E\Delta d = (1.708 \times 10^{4})(0.0360) = 615$ V.

Try it. An electron ($m = 9.11 \times 10^{-31}$ kg, $q = 1.60 \times 10^{-19}$ C) has an acceleration of 2.50×10^{15} m/s² between plates 0.0480 m apart. Find the potential difference between the plates.

1. $F = ma = (9.11 \times 10^{-31}\text{ kg})(2.50 \times 10^{15}\text{ m/s}^2) = 2.278 \times 10^{-15}$ N.

2. $E = F/q = 2.278 \times 10^{-15} \div (1.60 \times 10^{-19}) = 1.4238 \times 10^{4}$ N/C.

3. $\Delta V = E\Delta d = (1.4238 \times 10^{4}\text{ V/m})(0.0480\text{ m}) = 683$ V.

Slide 4 of 7 · The full two-direction problem

L is the **length of the plates** and Δd is the **gap between them**. Time is the **link** between the two directions. Horizontal motion is uniform: $t = \mathbf{L/v_x}$.

Vertically $\Delta d = v_i t + \frac{1}{2}at^2$, and the vertical starting speed v_i is **0**, so $\Delta d = \mathbf{\frac{1}{2}at^2}$, which gives $a = \mathbf{2\Delta d/t^2}$. Then run the chain: $F = \mathbf{ma}$, $E = \mathbf{F/q}$, $\Delta V = \mathbf{E\Delta d}$.

“Just reaches the far end of the other plate” means the particle starts beside one plate and covers the full gap Δd (the vertical distance) in the time t. This is the greatest voltage that lets the electron escape: a bigger voltage gives a bigger acceleration, and the electron would hit the plate **first**. Set up two columns, horizontal and vertical, and fill in what you know in each.

For the final speed, the vertical speed is $v_y = \mathbf{\sqrt{(2ad)}}$ (from $v^2 = v_i^2 + 2ad$ with $v_i = 0$, and d the vertical distance); combine it with v_x as two sides of a right triangle: $v = \mathbf{\sqrt{(v_x^2 + v_y^2)}}$, angle $\theta = \tan^{-1}(\mathbf{v_y/v_x})$, measured from the **horizontal**. Name the principles in order: uniform motion, then uniform acceleration.

Example. An electron enters horizontally at $v_x = 4.00 \times 10^6$ m/s beside one plate. The plates are L = 0.100 m long and Δd = 0.0400 m apart. It just reaches the far end of the other plate. Find the potential difference between the plates. ($m = 9.11 \times 10^{-31}$ kg, $q = 1.60 \times 10^{-19}$ C)

1. $t = L/v_x = 0.100 \div (4.00 \times 10^6) = 2.50 \times 10^{-8}$ s.

2. $\Delta d = \frac{1}{2}at^2$, so $a = 2\Delta d/t^2 = 2(0.0400) \div (2.50 \times 10^{-8})^2 = 1.28 \times 10^{14}$ m/s².

3. $F = ma = 1.166 \times 10^{-16}$ N, $E = F/q = 729$ N/C.

4. $\Delta V = E\Delta d = 29.2$ V.

Try it. An electron enters horizontally at 3.00×10^6 m/s beside one plate. The plates are 0.0800 m long and 0.0300 m apart, and it just reaches the far end of the other plate. Find the potential difference between the plates, and name the two physics principles you used, in order.

1. Horizontal: $t = L/v_x = 0.0800 \div (3.00 \times 10^6) = 2.667 \times 10^{-8}$ s.

2. Vertical: $\Delta d = \frac{1}{2}at^2$, so $a = 2(0.0300) \div (2.667 \times 10^{-8})^2 = 8.4354 \times 10^{13}$, which is 8.44×10^{13} m/s².

3. $F = ma = (9.11 \times 10^{-31})(8.4354 \times 10^{13}) = 7.685 \times 10^{-17}$ N. $E = F/q = 480$ N/C. $\Delta V = E\Delta d = (480)(0.0300) = 14.4$ V.

4. Principles in order: uniform motion, then uniform acceleration.

Slide 5 of 7 · Method: how to solve a two-direction problem

Step 1, **Split**: make an x column (along the plates, **constant speed**) and a y column (across the plates, **constant acceleration**).

Step 2, **List**: write what you know in each column. A sideways entry means the across speed at the start is **0**.

Step 3, **Time**: time is the **link** between the two columns. Get it from the column with enough numbers: $t = \mathbf{L \div v_x}$.

Step 4, **Link**: across the plates, $\Delta d = \mathbf{\frac{1}{2}at^2}$, $F = \mathbf{ma}$, $E = \mathbf{F \div q}$ and $\Delta V = \mathbf{E\Delta d}$. Use the chain forwards or backwards.

Step 5, **Combine**: $v_y = \mathbf{at}$. Then $v = \mathbf{\sqrt{(v_x^2 + v_y^2)}}$ and the angle = $\mathbf{\tan^{-1}(v_y \div v_x)}$ from the horizontal.

Check: v_x never changes and v_y grows with time, so the final speed must be **bigger than v_x**.

Example. An electron enters horizontally at $v_x = 4.00 \times 10^6$ m/s beside one plate. The plates are L = 0.100 m long and $\Delta d = 0.0400$ m apart, and it just reaches the far end of the other plate. Use the five steps to find the potential difference between the plates, then the final speed and the angle. ($m = 9.11 \times 10^{-31}$ kg, $q = 1.60 \times 10^{-19}$ C)

1. Step 1: x column along the plates at constant speed. y column across the plates with constant acceleration.

2. Step 2: x has L = 0.100 m and $v_x = 4.00 \times 10^6$ m/s. y has $\Delta d = 0.0400$ m and starting across speed 0.

3. Step 3: $t = L \div v_x = 0.100 \div (4.00 \times 10^6) = 2.50 \times 10^{-8}$ s.

4. Step 4: $a = 2\Delta d \div t^2 = 1.28 \times 10^{14}$ m/s². $F = ma = 1.166\ldots \times 10^{-16}$ N. $E = F \div q = 728.8\ldots$ N/C. $\Delta V = E\Delta d = 29.2$ V.

5. Step 5: $v_y = at = 3.20 \times 10^6$ m/s. $v = \sqrt{((4.00 \times 10^6)^2 + (3.20 \times 10^6)^2)} = 5.12 \times 10^6$ m/s. Angle $= \tan^{-1}(3.20 \div 4.00) = 38.7°$ from the horizontal.

Try it. NEW, for teacher review. An electron enters horizontally at 2.50×10^6 m/s beside one plate. The plates are 0.0600 m long and 0.0150 m apart, and it just reaches the far end of the other plate. Use the five steps to find the potential difference between the plates and the final speed.

1. Step 3: $t = L \div v_x = 0.0600 \div (2.50 \times 10^6) = 2.40 \times 10^{-8}$ s.

2. Step 4: $a = 2\Delta d \div t^2 = 2(0.0150) \div (2.40 \times 10^{-8})^2 = 5.2083 \times 10^{13}$ m/s^2.

3. $F = ma = 4.7448 \times 10^{-17}$ N. $E = F \div q = 296.55$ N/C. $\Delta V = E\Delta d = 296.55 \times 0.0150 = 4.45$ V.

4. Step 5: v_y = at = 1.25×10^6 m/s.

5. $v = \sqrt{((2.50 \times 10^6)^2 + (1.25 \times 10^6)^2)} = 2.80 \times 10^6$ m/s, at $\tan^{-1}(1.25 \div 2.50) = 26.6°$ from the horizontal.

Slide 6 of 7 · Where does the particle land?

A long numerical response is worth doing in order, with each result **labelled**: field, force, acceleration, time, distance.

Field from the voltage and the gap: $E = \mathbf{\Delta V/\Delta d}$. Acceleration with the particle's own charge and mass: $a = \mathbf{qE/m}$.

Vertical motion starts at zero vertical speed and covers the gap: $\Delta d = \mathbf{\frac{1}{2}at^2}$, so $t = \mathbf{\sqrt{(2\Delta d/a)}}$. Horizontal motion is constant speed, so the horizontal distance is $x = \mathbf{v_x t}$.

Check the units at the end, then record the answer as $\mathbf{a.bc \times 10^n}$ with its unit and three digits.

Example. An alpha particle ($q = 3.20 \times 10^{-19}$ C, $m = 6.65 \times 10^{-27}$ kg) enters horizontally at 2.00×10^5 m/s beside the top plate of a pair of plates 0.0260 m apart with 600 V across them. The top plate is positive and the bottom plate is negative, so the alpha particle is pushed toward the bottom plate and lands on it. Find the horizontal distance it travels. Record the answer as $a.bc \times 10^n$ m.

1. $E = \Delta V/\Delta d = 600 \div 0.0260 = 2.31 \times 10^4$ V/m.

2. $a = qE/m = (3.20 \times 10^{-19})(2.31 \times 10^4) \div (6.65 \times 10^{-27}) = 1.11 \times 10^{12}$ m/s².

3. Vertically, $\Delta d = \frac{1}{2}at^2$, so $t = \sqrt{(2 \times 0.0260 \div (1.11 \times 10^{12}))} = 2.164 \times 10^{-7}$ s.

4. $x = v_x t = (2.00 \times 10^5)(2.164 \times 10^{-7}) = 4.328 \times 10^{-2}$ m, which is 4.33×10^{-2} m. Record 4.33×10^{-2}.

Try it. An electron ($q = 1.60 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) enters horizontally at 5.00×10^6 m/s beside the top plate of a pair of plates 0.0200 m apart with 100 V across them. The top plate is negative and the bottom plate is positive, so the electron is pulled toward the bottom plate and lands on it. Find the horizontal distance it travels.

1. $E = \Delta V/\Delta d = 100 \div 0.0200 = 5.00 \times 10^3$ V/m.

2. $a = qE/m = (1.60 \times 10^{-19})(5.00 \times 10^3) \div (9.11 \times 10^{-31}) = 8.78 \times 10^{14}$ m/s².

3. Vertically, $\Delta d = \frac{1}{2}at^2$, so $t = \sqrt{(2 \times 0.0200 \div (8.78 \times 10^{14}))} = 6.7497 \times 10^{-9}$ s.

4. $x = v_x t = (5.00 \times 10^6)(6.7497 \times 10^{-9}) = 3.3749 \times 10^{-2}$ m, which is 3.37×10^{-2} m.

Slide 7 of 7 · Still charges: the oil-drop experiment

A drop that hangs still has **no net force**. The electric force **up** equals the weight **down**: qE = **mg**. So q = **mg ÷ E**. Find the field first: E = **ΔV ÷ Δd**.

Count the extra electrons with n = **q ÷ e**, where e = **1.60 × 10⁻¹⁹ C**. Electrons are counted, so n is a **whole number**. Use g = **9.81 m/s²**.

The field points from the positive plate to the negative plate. A drop pushed **against** the field is negative. A negative drop has **extra** electrons.

Millikan found that every measured charge was a whole-number multiple of **e**: q = **n × e**. A value that gives a **fraction** when divided by e cannot be right. Charge comes in **units**.

Example. An oil drop of mass 1.96×10^{-15} kg hangs still between plates 0.0100 m apart with 400 V across them. The top plate is positive. Find the charge on the drop and the number of extra electrons. (g = 9.81 m/s², e = 1.60×10^{-19} C)

1. E = ΔV ÷ Δd = 400 ÷ 0.0100 = 4.00×10^{4} N/C.

2. mg = $(1.96 \times 10^{-15})(9.81) = 1.9228... \times 10^{-14}$ N.

3. qE = mg, so q = mg ÷ E = $1.9228... \times 10^{-14} \div (4.00 \times 10^{4}) = 4.807 \times 10^{-19}$ C.

4. n = q ÷ e = $4.807 \times 10^{-19} \div (1.60 \times 10^{-19}) = 3.004...$, so 3 extra electrons. The force is up and the field is down, so the drop is negative.

Try it. NEW, for teacher review. A drop of mass 3.26×10^{-15} kg hangs still between plates 0.0200 m apart with 500 V across them. The top plate is positive. Find the charge on the drop and the number of extra electrons. (g = 9.81 m/s², e = 1.60×10^{-19} C)

1. E = ΔV ÷ Δd = 500 ÷ 0.0200 = 2.50×10^{4} N/C.

2. mg = $(3.26 \times 10^{-15})(9.81) = 3.1981... \times 10^{-14}$ N.

3. q = mg ÷ E = $3.1981... \times 10^{-14} \div (2.50 \times 10^{4}) = 1.279... \times 10^{-18}$ C.

4. n = q ÷ e = $1.2792... \times 10^{-18} \div (1.60 \times 10^{-19}) = 7.995...$, so 8 extra electrons.

B8 · Worked practice

Energy conservation and 2-D motion of charges | Questions 1-2 of 7

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass mp $= 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B8-N01. A charged ball ($m = 3.00 \times 10^{-4}$ kg) is released from rest where its electric potential energy is 5.0×10^{-6} J. At another point its electric potential energy is 8.0×10^{-7} J. Find its speed there.

1. Givens: $v_0 = 0$; $E_{p1} = 5.0 \times 10^{-6}$ J; $E_{p2} = 8.0 \times 10^{-7}$ J; $m = 3.00 \times 10^{-4}$ kg. Question: v.

2. $E_k = E_{p1} - E_{p2} = 4.2 \times 10^{-6}$ J.

3. $v = \sqrt{(2E_k/m)} = \sqrt{(2(4.2 \times 10^{-6}) \div (3.00 \times 10^{-4}))} = 0.167$ m/s.

4. Answer: 0.167 m/s

Easy · B8-N02. A proton enters a uniform field between horizontal plates moving parallel to the plates. Which statement describes its motion?

A. Constant velocity parallel to the plates and constant acceleration across them

B. Constant acceleration in both directions

C. Constant velocity in both directions

D. Velocity changes parallel to the plates only

1. Givens: entry parallel to the plates, field across the plates. Question: the motion.

2. The force is across the plates only, so the parallel component has no force and keeps constant speed; the perpendicular component accelerates.

3. Answer: A. Constant velocity parallel to the plates and constant acceleration across them

B8 · Worked practice

Energy conservation and 2-D motion of charges | Questions 3-4 of 7

Easy · B8-N03. Name the shape of the path of a charged particle that enters a uniform electric field at right angles to the field. Explain with one sentence about each direction.

1. Givens: entry at right angles to a uniform field. Question: path shape and reason.

2. The path is a parabola.

3. Parallel to the plates the speed is constant, and across the plates the acceleration is constant, as for a ball thrown horizontally.

Medium · B8-N04. An electron has an acceleration of 4.00×10^{15} m/s² between plates 0.0250 m apart. Find the potential difference between the plates.

1. Givens: $a = 4.00 \times 10^{15}$ m/s²; $\Delta d = 0.0250$ m; $q = 1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg. Question: ΔV.

2. $F = ma = qE$, so $E = ma/q$.

3. $E = 2.2775 \times 10^{4}$ V/m, then $\Delta V = E\Delta d = 569$ V.

4. Answer: 569 V

B8 · Worked practice

Energy conservation and 2-D motion of charges | Questions 5-6 of 7

Medium · B8-N05. An electron enters horizontally at 5.00×10^{6} m/s beside one plate. The plates are 0.120 m long. How long does the electron take to cross the plates?

1. Givens: $v_{0x} = 5.00 \times 10^{6}$ m/s; $\Delta x = 0.120$ m. Question: t.

2. No horizontal force, so $\Delta x = vt$.

3. $t = \Delta x \div v = 0.120 \div (5.00 \times 10^{6}) = 2.40 \times 10^{-8}$ s.

4. Answer: 2.40×10^{-8} s

Challenging · B8-N06. Use the electron from the previous question. It just reaches the far end of the other plate, so it moves 0.0500 m across the gap during that time. Find the potential difference between the plates.

1. Givens: $t = 2.40 \times 10^{-8}$ s; $\Delta y = 0.0500$ m across the plates; $q = 1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg; $v_{y0} = 0$. Question: ΔV.

2. Vertical motion: $\Delta y = \frac{1}{2}at^2$, so $a = 2\Delta y/t^2 = 1.74 \times 10^{14}$ m/s².

3. $F = ma$, $E = F/q$, $\Delta V = E\Delta d$ with $\Delta d = 0.0500$ m: $\Delta V = 49.4$ V.

4. Answer: 49.4 V

B8 · Worked practice

Challenging · B8-N07. In the previous two questions, name the principles you used in order. Give one principle that also describes the event but you did not need.

1. Givens: the two-step problem above. Question: order of the principles and one unused principle.

2. 1) Constant horizontal velocity gave the time. 2) Constant acceleration across the plates gave a. 3) Newton’s second law with F = qE gave E, then E = ΔV/Δd gave the voltage.

3. Energy conservation also describes the event, but it would need the final speed, which this question did not give.