B7 · Fields between plates

Completed notes

Use E = ΔV/Δd between parallel plates and find force, acceleration and speed of a charge.

Slide 1 of 5 · Radial and uniform fields

A field is a region where a **test charge** feels a force. The arrows show the force on a **positive** test charge.

Around a point charge the field is **radial**: arrows point **away from** a positive source and **toward** a negative one. Closer in, the arrows bunch up: a **stronger** field. The direction changes from place to place.

Between parallel plates the field is **uniform** (constant): parallel, evenly spaced arrows, the same at R, at S and between. The arrows point from the **positive** plate to the **negative** plate.

The force on a charge is F = **qE**, and F = ma links it to the acceleration. If E does not change along the path, neither does the **acceleration**.

Example. A small positive object is released from rest in each of two fields, a point-charge field (positive source) and the uniform field between plates, and nothing else acts on it. In each field, does its acceleration increase, decrease or stay the same as it moves, and what happens to its kinetic energy?

1. Point charge: the object is pushed away and the field is weaker the farther it goes, so the force and the acceleration decrease. It still speeds up the whole way.

2. Plates: the field is uniform, so the force and the acceleration stay constant.

3. In both cases the object loses electric potential energy and gains the same amount of kinetic energy.

Try it. Describe the direction and spacing of the field arrows (a) around a negative point charge and (b) between two plates when the top plate is positive. Sketch both.

1. (a) Radial arrows pointing toward the negative charge, bunched closer near it, so the field is stronger there.

2. (b) Parallel, evenly spaced arrows pointing down from the positive top plate to the negative bottom plate, so the field is uniform.

Slide 2 of 5 · The field between plates

A uniform field is the same everywhere between plates. A bigger **voltage** or a smaller **gap** makes a **stronger** field.

Formula: $E = \mathbf{\Delta V/\Delta d}$. E is the field strength, in **V/m** (size only). ΔV is the potential difference between the plates, in **volts**. Δd is the gap between the plates, in **metres**.

Direction: from the **positive** plate to the **negative** plate. V/m and **N/C** are the same unit. Multiply both sides by Δd: $\Delta V = \mathbf{E\Delta d}$.

Plates only. For one **point charge** use $E = kQ/r^2$, where Q is the source charge and r is the distance from it. Write each rearranged form out.

On a graph of E against 1/Δd, $E = \Delta V \times (1/\Delta d)$ matches y = mx: y is **E**, x is **1/Δd** and the slope is **ΔV**. Rewrite the equation to match y = mx first.

Example. Plates are 0.0150 m apart and connected to a 120 V battery. Find the field strength between the plates, then find the battery voltage needed for a field three times as strong with the same gap.

1. $E = \Delta V \div \Delta d = 120\ V \div 0.0150\ m = 8.00 \times 10^3\ V/m$.

2. Three times the field is 2.40×10^4 V/m. $\Delta V = E\Delta d = (2.40 \times 10^4\ V/m)(0.0150\ m) = 360\ V$.

3. Check: three times 120 V. Same gap, three times the field needs three times the voltage.

Try it. The battery voltage is kept the same while the plate gap is changed. The field is graphed against 1/Δd. The line passes through C = ($25\ m^{-1}$, 4.5×10^3 V/m) and D = ($100\ m^{-1}$, 18.0×10^3 V/m). What does the slope represent, and what is its value?

1. $E = \Delta V(1/\Delta d)$ matches y = mx, so the slope is the potential difference ΔV.

2. Slope = $(18.0 \times 10^3 - 4.5 \times 10^3)\ V/m \div (100 - 25)\ m^{-1} = 1.35 \times 10^4 \div 75 = 180\ V$.

Slide 3 of 5 · Force, acceleration and speed between plates

Force gives **acceleration**. Energy gives **speed**. Only the electric force acts, so ma = **qE**. From B4: F = qE, where q is the particle's charge.

Field: E = **ΔV/Δd**. Acceleration: a = **qE/m**. Speed from rest: qΔV = **½mv²**, so v = **√(2qΔV/m)**. The field is uniform, so a is **constant**.

If the question gives E and not ΔV, find ΔV = **EΔd** first. Use the electron's charge and mass from the data sheet. Keep every digit on your calculator and round **once**.

Use the particle's own q and m. A proton has the same size of charge as an electron but a much bigger **mass**. Two routes that agree are a good check. A speed stays well below **3×10^8 m/s**.

Example. An electron ($q = 1.60 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) starts from rest in a uniform field of 4.00×10^3 V/m. The plates are 0.0600 m apart. Find the final speed of the electron at the other plate.

1. $\Delta V = E\Delta d = (4.00 \times 10^3 \text{ V/m})(0.0600 \text{ m}) = 240$ V.

2. From rest: $q\Delta V = \frac{1}{2}mv^2$, so $v = \sqrt{(2q\Delta V/m)}$.

3. $v = \sqrt{(2 \times 1.60 \times 10^{-19} \times 240 \div (9.11 \times 10^{-31}))} = 9.18 \times 10^6$ m/s.

Try it. A proton ($q = 1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) starts from rest against the positive plate. The plates are 0.0150 m apart and the voltage is 300 V. Find its final speed using energy, then check the speed using the acceleration and the gap.

1. Energy: $v = \sqrt{(2q\Delta V/m)} = \sqrt{(2 \times 1.60 \times 10^{-19} \times 300 \div (1.67 \times 10^{-27}))} = 2.40 \times 10^5$ m/s.

2. Force: $E = \Delta V/\Delta d = 2.00 \times 10^4$ V/m. $a = qE/m = 1.92 \times 10^{12}$ m/s².

3. $v = \sqrt{(2a\Delta d)} = 2.40 \times 10^5$ m/s. The two routes agree.

Slide 4 of 5 · Method: parallel-plate problems

Step 1: find the field. E = **ΔV/Δd**, with the gap in **metres**.

Step 2: find the field direction. It points from the **positive** plate to the **negative** plate.

Step 3: find the force. F = **qE**, using the size of q. A positive charge is pushed **along** the field. A negative charge is pushed **against** it.

Step 4: find the acceleration. a = **F ÷ m**, in the same direction as the force. Step 5: find the speed. From rest, qΔV = **½mv²**, or check with v = **√(2aΔd)**. Check: a speed stays well below **3 × 10⁸ m/s**.

Example. An alpha particle ($q = 3.20 \times 10^{-19}$ C, $m = 6.65 \times 10^{-27}$ kg) starts from rest against the positive plate. The gap is 0.0200 m and the battery is 175 V. Find its acceleration and its speed at the other plate.

1. Step 1: $E = \Delta V/\Delta d = 175 \div 0.0200 = 8.75 \times 10^{3}$ V/m. Step 2: the field points from the positive plate to the negative plate.

2. Step 3: $F = qE = (3.20 \times 10^{-19})(8.75 \times 10^{3}) = 2.800 \times 10^{-15}$ N, along the field because the charge is positive.

3. Step 4: $a = F \div m = 2.800 \times 10^{-15} \div (6.65 \times 10^{-27}) = 4.2105\ldots \times 10^{11}$ m/s², which is 4.21×10^{11} m/s².

4. Step 5: $v = \sqrt{(2a\Delta d)} = \sqrt{(2 \times 4.2105\ldots \times 10^{11} \times 0.0200)} = 1.30 \times 10^{5}$ m/s. The energy route gives the same speed.

Try it. An electron ($q = 1.60 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) starts from rest against the negative plate. The gap is 0.0300 m and the battery is 450 V. Find its acceleration and its speed at the other plate. Write each step number beside your work.

1. Step 1: $E = 450 \div 0.0300 = 1.50 \times 10^{4}$ V/m. Step 2: the field points from the positive plate to the negative plate, toward the electron’s starting plate.

2. Step 3: $F = qE = (1.60 \times 10^{-19})(1.50 \times 10^{4}) = 2.40 \times 10^{-15}$ N. The charge is negative, so the force is against the field, toward the positive plate.

3. Step 4: $a = 2.40 \times 10^{-15} \div (9.11 \times 10^{-31}) = 2.6345\ldots \times 10^{15}$ m/s².

4. Step 5: $v = \sqrt{(2 \times 2.6345\ldots \times 10^{15} \times 0.0300)} = 1.26 \times 10^{7}$ m/s. Check: well below 3×10^{8} m/s.

Slide 5 of 5 · Not starting from rest

If a particle is already moving when it enters (look for "**enters at**"), it has initial kinetic energy ½mv_i², and the field adds more or takes some away.

Find ΔV = **EΔd**. Find the initial kinetic energy from the entering speed. Find the energy gained or lost: **qΔV**.

An electron moving toward the + plate: unlike charges **attract**, so it **speeds up**. Ek final = Ek initial **+ qΔV**. A proton moving toward the + plate: like charges **repel**, so it **slows down**. Ek final = Ek initial **− qΔV**.

Then v_f = **√(2Ek_final/m)**. Check: a charge that speeds up ends **faster**, and a slowed charge ends **slower**, than its starting speed.

Example. An electron ($q = 1.60 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) enters the gap at 2.00×10^{6} m/s toward the positive plate. The field is 3.00×10^{3} V/m and the gap is 0.0500 m. Find its speed at the positive plate.

1. $\Delta V = E\Delta d = (3.00 \times 10^{3}\ \text{V/m})(0.0500\ \text{m}) = 150$ V.

2. Initial kinetic energy = $\frac{1}{2}(9.11 \times 10^{-31})(2.00 \times 10^{6})^2 = 1.82 \times 10^{-18}$ J. Energy gained = $q\Delta V = 2.40 \times 10^{-17}$ J.

3. Final kinetic energy = 2.58×10^{-17} J.

4. $v = \sqrt{(2 \times 2.58 \times 10^{-17} \div (9.11 \times 10^{-31}))} = 7.53 \times 10^{6}$ m/s, which is faster than 2.00×10^{6} m/s.

Try it. A proton ($q = 1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) enters the gap moving at 4.0×10^{5} m/s toward the positive plate, which repels it. The potential difference is 400 V. Find its speed at the positive plate, and say how you know the answer makes sense.

1. Initial kinetic energy = $\frac{1}{2}(1.67 \times 10^{-27})(4.0 \times 10^{5})^2 = 1.34 \times 10^{-16}$ J.

2. The positive plate repels the proton, so it gives up $q\Delta V = (1.60 \times 10^{-19})(400) = 6.40 \times 10^{-17}$ J.

3. Final kinetic energy = $1.336 \times 10^{-16} - 6.40 \times 10^{-17} = 6.96 \times 10^{-17}$ J.

4. $v = \sqrt{(2 \times 6.96 \times 10^{-17} \div (1.67 \times 10^{-27}))} = 2.89 \times 10^{5}$ m/s.

5. Check: it is less than 4.0×10^{5} m/s, as it should be for a slowed charge.

B7 · Worked practice

Fields between plates | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass $m_p = 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B7-N01. Two parallel plates 0.0300 m apart are connected to a 150 V battery. Find the electric field strength between them in V/m.

1. Givens: $\Delta d = 0.0300$ m; $\Delta V = 150$ V. Question: E.

2. $E = \Delta V \div \Delta d = 150 \div 0.0300 = 5000$ V/m (5000 V/m).

3. Answer: 5000 V/m

Easy · B7-N02. The gap between two plates is halved while the battery voltage stays the same. What happens to the field strength between them?

A. It doubles.
B. It halves.
C. It stays the same.
D. It becomes four times larger.

1. Givens: ΔV constant; $\Delta d \div 2$. Question: change in E.

2. $E = \Delta V/\Delta d$, so dividing Δd by 2 multiplies E by 2.

3. Answer: A. It doubles.

B7 · Worked practice

Fields between plates | Questions 3-4 of 8

Easy · B7-N03. A positive charge is released from rest at different places between large parallel plates. How does its acceleration compare from place to place?

A. It is larger near the positive plate.
B. It is larger near the negative plate.
C. It increases as the charge speeds up.
D. It is the same everywhere, because the field is uniform.

1. Givens: a uniform field. Question: how a changes.

2. $F = qE$ with the same E everywhere, so F and $a = F/m$ are constant.

3. Answer: D. It is the same everywhere, because the field is uniform.

Medium · B7-N04. An electron starts from rest in a uniform field of 2500 V/m across plates 0.08 m apart. Find its final speed.

1. Givens: $E = 2500$ V/m; $\Delta d = 0.08$ m; $q = 1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg; $v_0 = 0$. Question: v.

2. First the potential difference: $\Delta V = E\Delta d = (2500)(0.08) = 200$ V.

3. Then $q\Delta V = \frac{1}{2}mv^2$: $v = \sqrt{(2q\Delta V/m)} = 8.38 \times 10^6$ m/s.

4. Answer: 8.38×10^6 m/s

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Fields between plates | Questions 5-6 of 8

Medium · B7-N05. A proton starts at rest between plates 0.0400 m apart with a potential difference of 800 V. Use e = 1.60 × 10^{-19} C and proton mass mp = 1.67 × 10^{-27} kg. Find its acceleration in m/s².

1. Givens: ΔV = 800 V; Δd = 0.0400 m; q = 1.60 × 10^{-19} C; m = 1.67 × 10^{-27} kg. Question: a.

2. E = ΔV/Δd = 800 ÷ 0.0400 = 2.00 × 10^{4} V/m.

3. F = qE = 3.20 × 10^{-15} N.

4. a = F ÷ m = 1.92 × 10^{12} m/s².

5. Answer: 1.92 × 10^{12} m/s²

Medium · B7-N06. Using the proton in the previous question, find its final speed when it reaches the far plate.

1. Givens: a = 1.92 × 10^{12} m/s²; Δd = 0.0400 m; v_0 = 0. Question: v.

2. $v^2 = v_0^2 + 2ad$.

3. v = √(2(1.92 × 10^{12})(0.0400)) = 3.92 × 10^{5} m/s.

4. Check with energy: √(2qΔV/m) gives the same value.

5. Answer: 3.92 × 10^{5} m/s

B7 · Worked practice

Fields between plates | Questions 7-8 of 8

Challenging · B7-N07. An electron enters the gap at 1.0×10^{6} m/s toward the positive plate. The plates are 0.020 m apart with 3.0×10^{3} V across them. Find its speed at the positive plate.

1. Givens: $v_0 = 1.0 \times 10^{6}$ m/s; $\Delta d = 0.020$ m; $\Delta V = 3.0 \times 10^{3}$ V; $q = 1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg. Question: v.

2. $a = q\Delta V/(m\Delta d) = 2.6 \times 10^{16}$ m/s².

3. $v = \sqrt{(v_0^2 + 2ad)} = \sqrt{(1.0 \times 10^{12} + 1.05 \times 10^{15})} = 3.2 \times 10^{7}$ m/s.

4. The electron started with speed, so we keep v_0^2 in the equation.

5. Answer: 3.2×10^{7} m/s

Challenging · B7-N08. A graph of E against $1/\Delta d$ for a fixed battery is a straight line through the origin with slope 150 V. What does the slope represent, and what happens to E if the gap is doubled?

1. Givens: E vs $1/\Delta d$, slope 150 V. Question: meaning of the slope; effect of doubling the gap.

2. $E = \Delta V(1/\Delta d)$ matches $y = mx$, so the slope is the battery voltage, 150 V.

3. Doubling Δd halves $1/\Delta d$, so E is halved.