B6 · Potential difference: energy per charge

Completed notes

Explain potential difference as energy per charge, use $\Delta E = q\Delta V$, and use the electron volt.

Slide 1 of 7 · Work that stores energy between plates

A positive particle against the **negative** plate is where it wants to be, so we call its electric potential energy (written Ep) **zero** there, like a ball at the bottom of a hill.

To carry it toward the other plate something outside must push, because the plates pull and push it **back**. Our push does **work**.

The speed stays constant, so none of the work becomes **kinetic energy**. **All** the work is stored as electric potential energy: $\Delta Ep = 6.0 \times 10^{-16}$ J in the example, where Δ means **“change in”**. When released, Ep decreases and the same amount becomes kinetic energy, written Ek.

The positive plate **pushes** a positive charge away and the negative plate **pulls** it in.

Example. A positive particle with $q = 4.0 \times 10^{-17}$ C is carried at constant speed from the negative top plate (A) to B, gaining 6.0×10^{-16} J of electric potential energy. It is released from rest and gravity is too small to matter. Which way does it move, and how much kinetic energy does it have when it reaches the top plate?

1. The positive plate repels the particle and the negative plate attracts it, so the force at B points up and the particle moves up toward the negative top plate.

2. As it moves, electric potential energy turns into kinetic energy. With no other losses, all 6.0×10^{-16} J becomes kinetic energy at the top plate.

Try it. The same positive particle is held at B, with 6.0×10^{-16} J of electric potential energy, and is carried at constant speed back to the negative top plate at A. Does its electric potential energy increase or decrease, by how much, and what is its potential energy at A?

1. The field pushes the positive particle toward the negative plate, so the field does positive work as it moves from B to A.

2. The stored energy decreases by 6.0×10^{-16} J ($\Delta Ep = -6.0 \times 10^{-16}$ J).

3. At A, $Ep = 6.0 \times 10^{-16}$ J $- 6.0 \times 10^{-16}$ J $= 0$, which is the zero point we chose.

Slide 2 of 7 · Potential difference: energy per charge

A bigger charge stores more energy, so divide out the charge. Electric potential difference is $\Delta V = \mathbf{\Delta E \div q}$: ΔE is the **energy change** (J) and q is the **size of the charge** (C).

Its unit is **J/C** (joules per coulomb). ΔV belongs to the **plates**, not the particle: a charge three times as big stores three times the energy, and $\Delta E \div q$ gives the **same** ΔV.

Rearrange for the unknown: $\Delta E = \mathbf{q\Delta V}$. Put in the size of the charge, then ask which way it moves: moving toward the plate that attracts it means it **gains kinetic energy**. If the plates do not change, ΔV stays the **same** for a new charge.

On a graph of ΔE against q, $\Delta E = q\Delta V$ matches $y = mx$, so the slope is **ΔV**. The line passes through the **origin**, because zero charge stores zero energy. Check powers of ten, since the charges are very small.

Example. On other plates, a charge $q = 6.0 \times 10^{-18}$ C stores 1.8×10^{-16} J when moved from A to B. Find the potential difference between A and B, then find the energy stored when a charge of 1.5×10^{-17} C is moved between the same two points.

1. $\Delta V = \Delta E \div q = (1.8 \times 10^{-16}\ \text{J}) \div (6.0 \times 10^{-18}\ \text{C}) = 30\ \text{J/C}$.

2. The plates are the same, so $\Delta V = 30$ J/C. $\Delta E = q\Delta V = (1.5 \times 10^{-17}\ \text{C})(30\ \text{J/C}) = 4.5 \times 10^{-16}$ J.

Try it. The same plates move several charges from A to B, and the energy stored is graphed against the charge. The line passes through $C = (2.0 \times 10^{-18}\ \text{C}, 0.60 \times 10^{-16}\ \text{J})$ and $D = (8.0 \times 10^{-18}\ \text{C}, 2.40 \times 10^{-16}\ \text{J})$. What does the slope represent, and what is its value?

1. Match $\Delta E = q\Delta V$ to $y = mx$. ΔE is vertical, q is horizontal, so the slope is the potential difference ΔV.

2. Slope $= (2.40 \times 10^{-16} - 0.60 \times 10^{-16})\ \text{J} \div (8.0 \times 10^{-18} - 2.0 \times 10^{-18})\ \text{C} = 1.8 \times 10^{-16} \div (6.0 \times 10^{-18}) = 30$ J/C.

3. The line passes through the origin because zero charge stores zero energy.

Slide 3 of 7 · Method: how to solve an energy-per-charge problem

Step 1: name the **unknown**. Is it ΔE, q or ΔV? Write it before any number.

Step 2: write the equation with the unknown alone: ΔV = **ΔE/q**, ΔE = **qΔV** or q = **ΔE/ΔV**.

Step 3: put in the **sizes**: the size of the charge in coulombs and the energy in joules. Then ask which way it moves: toward the plate that attracts it, it gains **kinetic energy**.

Step 4: calculate with units. Joules per coulomb is the **volt**. Check: a bigger charge stores **more** energy.

Example. A charge of 6.0×10^{-18} C stores 1.8×10^{-16} J when it is moved from A to B. A charge of 1.5×10^{-17} C is then moved between the same two points. Find the potential difference, then the energy stored for the new charge.

1. Steps 1 and 2: part one wants ΔV, so ΔV = ΔE ÷ q. Part two wants ΔE, so ΔE = qΔV.

2. Step 3: ΔV = $(1.8 \times 10^{-16}$ J) ÷ $(6.0 \times 10^{-18}$ C). Then ΔE = $(1.5 \times 10^{-17}$ C) × ΔV, using the sizes of the charges.

3. Step 4: ΔV = 30 J/C, so ΔE = $(1.5 \times 10^{-17})(30) = 4.5 \times 10^{-16}$ J. The unit J/C is the volt, so ΔV = 30 V.

Try it. A charge of 2.5×10^{-17} C stores 1.0×10^{-15} J when it is moved between two plates. Find the potential difference between the plates, then the energy stored when a charge of 8.0×10^{-18} C is moved between the same plates.

1. Steps 1 and 2: first ΔV = ΔE ÷ q, then ΔE = qΔV.

2. Step 3: ΔV = $(1.0 \times 10^{-15}$ J) ÷ $(2.5 \times 10^{-17}$ C).

3. Step 4: ΔV = 40 J/C. Then ΔE = $(8.0 \times 10^{-18}$ C)(40 J/C) = 3.2×10^{-16} J.

4. Check: the smaller charge stores less energy than the first charge.

Slide 4 of 7 · The volt: one joule per coulomb

One volt is one **joule per coulomb**: 1 V = **1 J/C**. “Voltage” is just **potential difference**. A battery label such as 9.0 V means ΔV = **9.0 V** in an equation. Unlike the field E, ΔV has **no direction**.

A 9.0 V battery gives **9.0 J** to each coulomb that passes through it. A battery does not store charge to hand out; it supplies **energy** for each coulomb already in the wires.

Pick the form that matches the unknown: energy from charge, ΔE = **qΔV**; charge from energy, q = **ΔE ÷ ΔV**. The unit V can always be swapped for **J/C**.

Example. A battery is labelled 9.0 V and is connected to a heating coil. A charge of 4.0 C flows through the coil. Find the energy delivered.

1. ΔE = qΔV = (4.0 C)(9.0 V) = 36 J.

2. That energy shows up as heat in the coil.

Try it. A 6.0 V battery is connected to a lamp. How much energy is delivered when 2.5 C of charge passes through the lamp, and how much charge must pass to deliver 90 J?

1. ΔE = qΔV = (2.5 C)(6.0 V) = 15 J.

2. q = ΔE ÷ ΔV = 90 J ÷ 6.0 V = 15 C.

3. Check: 6.0 J per coulomb, so 90 J needs many coulombs.

Slide 5 of 7 · A charge accelerated through a voltage

Voltage supplies energy and the particle turns it into speed. Rearrange $\Delta V = \Delta E/q$ to $\Delta E = q\Delta V$. Kinetic energy is $E_k = \frac{1}{2}mv^2$; the small i means **initial** and f means **final**. Only the electric force changes the energy here, so all of $q\Delta V$ becomes kinetic energy: $q\Delta V = \mathbf{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$.

The initial term drops out only when the problem says **“from rest”** ($v_i = \mathbf{0}$). Do not drop it automatically.

From rest: $v = \mathbf{\sqrt{(2 \times E_k \div m)}}$, where $E_k = q\Delta V$. If it is already moving, find the **start** energy $\frac{1}{2}mv_i^2$ and the **gain** $q\Delta V$. Final E_k = start + gain, then solve for v.

Use the particle’s own q and m (electron, proton and alpha particle differ). Check: a particle sped up by the voltage must end **faster** than it started.

Example. An electron starts from rest at the negative plate of an electron gun and is accelerated through $\Delta V = 1.80 \times 10^{3}$ V. With $q = 1.60 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg, find its speed at the positive plate.

1. $q\Delta V = (1.60 \times 10^{-19}\ \text{C})(1.80 \times 10^{3}\ \text{V}) = 2.88 \times 10^{-16}$ J.

2. It starts from rest, so $\frac{1}{2}mv^2 = 2.88 \times 10^{-16}$ J.

3. $v = \sqrt{(2 \times 2.88 \times 10^{-16} \div (9.11 \times 10^{-31}))} = 2.51 \times 10^{7}$ m/s.

Try it. An electron enters the gap at $v_i = 1.50 \times 10^{7}$ m/s and is accelerated through $\Delta V = 6.00 \times 10^{2}$ V. Find its final speed. Use $q = 1.60 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg.

1. Initial energy: $\frac{1}{2}mv_i^2 = \frac{1}{2}(9.11 \times 10^{-31})(1.50 \times 10^{7})^2 = 1.024 \times 10^{-16}$ J.

2. Energy gained: $q\Delta V = (1.60 \times 10^{-19})(6.00 \times 10^{2}) = 9.600 \times 10^{-17}$ J.

3. Final energy $= 1.024 \times 10^{-16} + 9.600 \times 10^{-17} = 1.984 \times 10^{-16}$ J.

4. $v_f = \sqrt{(2 \times 1.984 \times 10^{-16} \div (9.11 \times 10^{-31}))} = 2.09 \times 10^{7}$ m/s.

5. Check: faster than 1.50×10^{7} m/s. (Treating it as starting from rest gives 1.45×10^{7} m/s, a typical wrong choice.)

Slide 6 of 7 · Method: how to find the speed of a charge accelerated through a voltage

Step 1: write the energy rule **$qΔV = ½mv_f^2 − ½mv_i^2$**. The energy gained from the voltage becomes **kinetic energy**.

Step 2: find the energy **gained**: $qΔV$, with the size of the charge in coulombs and the voltage in volts. The answer is in **joules**.

Step 3: find the **starting** energy. “From rest” means $½mv_i^2 = \mathbf{0}$. If it is already moving, work out $½mv_i^2$ with the particle’s own **mass**.

Step 4: final Ek = start + **gain**. Then $v = \mathbf{\sqrt{(2 \times Ek \div m)}}$. Check: the unit is **m/s**, and a particle sped up by the voltage ends **faster** than it began.

Example. An electron starts from rest and crosses a gap with a voltage of 1.80×10^3 V. With $q = 1.60 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg, find its speed at the end.

1. Step 1: $qΔV = ½mv_f^2 − ½mv_i^2$.

2. Step 2: $qΔV = (1.60 \times 10^{-19}\ \text{C})(1.80 \times 10^3\ \text{V}) = 2.88 \times 10^{-16}$ J. Step 3: it starts from rest, so the starting energy is 0.

3. Step 4: final Ek = 2.88×10^{-16} J. $v = \sqrt{(2 \times 2.88 \times 10^{-16} \div (9.11 \times 10^{-31}))} = 2.5145\ldots \times 10^7$ m/s, so $v = 2.51 \times 10^7$ m/s.

Try it. A proton starts from rest and is accelerated through 2.00×10^3 V. With $q = 1.60 \times 10^{-19}$ C and $m = 1.67 \times 10^{-27}$ kg, find its final speed.

1. Step 1: $qΔV = ½mv_f^2 − ½mv_i^2$.

2. Step 2: $qΔV = (1.60 \times 10^{-19}\ \text{C})(2.00 \times 10^3\ \text{V}) = 3.20 \times 10^{-16}$ J. Step 3: from rest, so the starting energy is 0.

3. Step 4: $v = \sqrt{(2 \times 3.20 \times 10^{-16} \div (1.67 \times 10^{-27}))} = 6.1906\ldots \times 10^5$ m/s, so $v = 6.19 \times 10^5$ m/s.

4. Check: the unit is m/s, and the proton has much more mass than an electron, so it ends slower than 2.51×10^7 m/s.

Slide 7 of 7 · The electron volt

The electron volt (eV) is a unit of **energy**, not of voltage. One eV is the energy an electron gains when accelerated through **one volt**.

1 eV = $q\Delta V$ = $(1.60 \times 10^{-19}\ \text{C})(1\ \text{V})$ = **1.60×10^{-19} J** (data sheet).

A charge of n electrons gains **n eV** for each volt: an electron through 24.0 V gains **24.0** eV, an alpha particle (charge 2e) through 15.0 V gains **30.0** eV.

To switch units: eV × 1.60×10^{-19} = **joules**, and joules ÷ (1.60×10^{-19}) = **eV**. The equation $\Delta E = q\Delta V$ still needs joules, coulombs and volts. Read the unit in the question first and finish in it.

Example. An alpha particle (charge 3.20×10^{-19} C) is accelerated from rest through 45.0 V. Find the kinetic energy it gains, first in electron volts and then in joules.

1. Its charge is 2e, so each volt gives it 2 eV: 2 × 45.0 V = 90.0 eV.

2. In joules: 90.0 eV × 1.60×10^{-19} J/eV = 1.44×10^{-17} J.

3. Check: $q\Delta V = (3.20 \times 10^{-19}\ \text{C})(45.0\ \text{V}) = 1.44 \times 10^{-17}$ J. The two routes agree.

Try it. An electron enters the gap at 1.0×10^{6} m/s and is accelerated through 80.0 V. Find its final kinetic energy in electron volts. Record the answer as $a.bc \times 10^{n}$ eV. Use $m = 9.11 \times 10^{-31}$ kg.

1. Gain: an electron through 80.0 V gains 80.0 eV.

2. Initial energy: $\frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31}\ \text{kg})(1.0 \times 10^{6}\ \text{m/s})^2 = 4.555 \times 10^{-19}$ J, and $4.555 \times 10^{-19} \div (1.60 \times 10^{-19}) = 2.847$ eV.

3. Final energy = 80.0 eV + 2.847 eV = 82.847 eV, which rounds to 82.8 eV.

4. Record 8.28×10^{1} eV. Round only at the end.

B6 · Worked practice

Potential difference: energy per charge | Questions 1-2 of 7

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}\ N \cdot m^2/C^2$, $e = 1.60 \times 10^{-19}\ C$, electron mass $m_e = 9.11 \times 10^{-31}\ kg$, proton mass $mp = 1.67 \times 10^{-27}\ kg$, $g = 9.81\ m/s^2$.

Easy · B6-N01. A charge of 5.0×10^{-18} C is moved between two points, and 1.5×10^{-16} J of energy is stored. Find the potential difference.

1. Givens: $q = 5.0 \times 10^{-18}$ C; $\Delta E = 1.5 \times 10^{-16}$ J. Question: ΔV.

2. $\Delta V = \Delta E \div q$.

3. $\Delta V = 1.5 \times 10^{-16} \div (5.0 \times 10^{-18}) = 30.0$ V.

4. Answer: 30 V

Easy · B6-N02. A 12 V battery drives 8.0 C of charge through a motor. How much energy does the battery deliver, in joules?

1. Givens: $\Delta V = 12$ V; $q = 8.0$ C. Question: ΔE.

2. $\Delta E = q\Delta V = (8.0)(12) = 96$ J.

3. Answer: 96 J

B6 · Worked practice

Potential difference: energy per charge | Questions 3-4 of 7

Easy · B6-N03. The potential difference between two points is 30 V. A second charge, twice as large, is moved between the same points. What is the new potential difference?

A. 60 V, because the charge doubled
B. 15 V, because q is in the denominator
C. Zero, because the energy cancels
D. 30 V, because ΔV belongs to the points and not to the charge

1. Givens: ΔV = 30 V for the first charge; the charge doubles. Question: the new ΔV.

2. The energy doubles, but ΔV = ΔE/q stays the same because both double.

3. Answer: D. 30 V, because ΔV belongs to the points and not to the charge

Medium · B6-N04. An electron is accelerated from rest through 3500 V. Use e = 1.60 × 10^{-19} C and electron mass m_e = 9.11 × 10^{-31} kg. Find its final speed.

1. Givens: v_0 = 0; ΔV = 3500 V; q = 1.60 × 10^{-19} C; m = 9.11 × 10^{-31} kg. Question: v.

2. $q\Delta V = \frac{1}{2}mv^2$, so $v = \sqrt{(2q\Delta V/m)}$.

3. $v = \sqrt{(2(1.60 \times 10^{-19})(3500) \div (9.11 \times 10^{-31}))} = 3.51 \times 10^7$ m/s.

4. This is well below the speed of light, so the formula $\frac{1}{2}mv^2$ is reasonable.

5. Answer: 3.51 × 10^7 m/s

B6 · Worked practice

Potential difference: energy per charge | Questions 5-6 of 7

Medium · B6-N05. A particle with charge 4.80×10^{-19} C is accelerated from rest through 25.0 V. Find the kinetic energy it gains, in electron volts.

1. Givens: $q = 4.80 \times 10^{-19}$ C $= 3e$; $\Delta V = 25.0$ V. Question: the energy gained in eV.

2. The charge is 3e, so it gains 3 eV for each volt.

3. $3 \times 25.0 = 75.0$ eV.

4. Answer: 75.0 eV

Challenging · B6-N06. An electron enters a gap moving at 2.0×10^{7} m/s and is accelerated through 4.00×10^{2} V. Find its final speed.

1. Givens: $v_0 = 2.0 \times 10^{7}$ m/s; $\Delta V = 4.00 \times 10^{2}$ V; $q = 1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg. Question: v.

2. The electron already has kinetic energy: $\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + q\Delta V$.

3. $v = \sqrt{(v_0^2 + 2q\Delta V/m)} = \sqrt{(4.000 \times 10^{14} + 1.405 \times 10^{14})} = 2.32 \times 10^{7}$ m/s. Keep the full values on your calculator.

4. The speed rises but is not more than the starting speed squared plus the gain, so it is reasonable. Do not add the speeds directly.

5. Answer: 2.32×10^{7} m/s

B6 · Worked practice

Potential difference: energy per charge | Questions 7-7 of 7

Challenging · B6-N07. An electron and a proton are each accelerated from rest through the same 100 V. Which gains more kinetic energy, and which ends with the greater speed? Explain.

1. Givens: same $\Delta V = 100$ V; equal size of charge e; masses 9.11×10^{-31} kg and 1.67×10^{-27} kg. Question: compare energy and speed.

2. Both gain the same energy, $q\Delta V = 100$ eV, because the charge magnitude and voltage are equal.

3. From $\frac{1}{2}mv^2 = q\Delta V$, $v = \sqrt{(2q\Delta V/m)}$. The electron has the smaller mass, so it ends with the greater speed.