B5 · Reading straight-line graphs

Student notes - follow the slides, fill the blanks, work the questions

Find a slope with its unit, match a graph to y = mx + b, and straighten a curve.

Name: _____ Date: _____

Slide 1 of 8 · Two variables and a line of best fit

A fair test changes one thing and measures one thing. The _____ variable is the one you change. The _____ variable is the one you measure. Everything else is held _____.

The manipulated variable goes on the _____ axis. The responding variable goes on the _____ axis. Spring test: the stretch (m) is _____. The force (N) is _____.

Points never fall exactly on a line. Every reading has a small _____. A line of best fit follows the _____.

Read values from the _____, never from single data points. Choose two points on the line that are _____, so the error is a small share of the answer.

Example. A student changes the source charge Q and measures the electric field strength E with a probe. The probe stays 0.500 m from the source in air. Which quantity is manipulated, which is responding, which is held fixed, and which one goes on the horizontal axis?

Work:

Answer: _____

Try it. A student draws a line of best fit through six scattered points. To find the slope she reads two of the data points that are close together. Give two things she should change.

Work:

Answer: _____

Slide 2 of 8 · Find the slope from the line

Slope = _____. Rise is how far the line goes _____. Run is how far it goes _____. Take B minus A for both, in the _____. F_B is the force at B. x_A is the extension at A.

Slope = _____. It shows how much the vertical quantity changes for each _____ of the horizontal quantity.

The slope unit is the _____ ÷ the _____. Here F is in N and x is in m, so the slope is in _____. A slope without its unit is incomplete.

Convert the run to _____. Keep the power of ten. A slope of 250 or 0.25 instead of 25.0 N/m means the _____ was lost.

Example. Points A (2.0×10^{-2} m, 1.00 N) and B (10.0×10^{-2} m, 3.00 N) lie on the line of best fit of spring force F against extension x. Find the slope.

Work:

Answer: _____

Try it. A different spring's line of best fit passes through C = (3.0×10^{-2} m, 1.40 N) and D = (9.0×10^{-2} m, 3.80 N). Find the slope, including its unit. Show the rise, the run and the unit check.

Work:

Answer: _____

Sketch (draw and label the diagram):

Slide 3 of 8 · Match the physics equation to y = mx + b

A straight line follows y = _____. m is the _____. b is the _____, where the line crosses the vertical axis (x = _____). Hooke’s law is F = kx. F is the force (N). x is the extension (m). k is the spring constant (N/m).

Match the symbols one at a time. The vertical-axis quantity plays _____. The horizontal-axis quantity plays _____. The number multiplying x is the _____. The term that stands alone is the _____.

In F = kx, F plays y and x plays x. So the slope is the _____. The law has _____ b term, so a 0.50 N intercept is not part of the law.

Sometimes the intercept is real physics. In $v = v_0 + at$, a is the constant acceleration (m/s^2) and v_0 is the starting speed (m/s). The term with no t is the _____. Always ask if the physics equation has a _____.

Example. A cart’s speed v is plotted against time t, and the line of best fit has the equation $v = v_0 + at$, where a is a constant acceleration. Using y = mx + b, what does the slope represent and what does the vertical intercept represent?

Work:

Answer: _____

Try it. A graph of the electric field strength E against the source charge Q, measured at a fixed distance r from a point source, matches $E = kQ/r^2$. Using y = mx + b, what does the slope equal, and what should the intercept be?

Work:

Answer: _____

Slide 4 of 8 · Predict with the whole line, intercept included

Predict with the whole line: y = _____, intercept included. The spring line is F = (25.0 N/m)x + 0.50 N.

To find x from F, write the equation with F in it. _____ first. Then _____. For F = 1.20 N: 0.70 = 25.0x, so x = _____. F = kx alone gives 0.0480 m and leaves out the 0.50 N.

The line is in SI units, so convert first: 6.0 cm = _____ m. Forgetting to convert is a classic wrong choice.

Inside the measured range is _____. It is reliable. Past the range is _____. It is less reliable, because the line may not stay straight.

Example. The spring line is F = (25.0 N/m)x + 0.50 N, from measured extensions up to 12×10^{-2} m. The extension is 6.0 cm. Find the predicted force.

Work:

Answer: _____

Try it. The same line is F = (25.0 N/m)x + 0.50 N. A force of 2.30 N is applied. Find the extension this experiment predicts, in metres.

Work:

Answer: _____

Slide 5 of 8 · Method: how to get slope and constants from a graph

Step 1: pick _____ on the line. Use grid crossings, far apart. Use line points, not _____.

Step 2: _____ = rise ÷ run. Keep the power of ten. The unit is the vertical unit ÷ the _____.

Step 3: match y = mx + b. Step 4: say what the slope and the _____ mean, with units. Does the physics equation have a b term?

Step 5: predict with the _____. Check: the unit and the size fit the picture.

Example. The spring line is F = (25.0 N/m)x + 0.50 N. A and B lie on it at (2.0 × 10^{-2} m, 1.00 N) and (10.0 × 10^{-2} m, 3.00 N). Find the spring constant, then the extension for a force of 1.20 N. Write each step number beside your work.

Work:

Answer: _____

Try it. A line of best fit for another spring passes through (2.0 × 10^{-2} m, 1.50 N) and (8.0 × 10^{-2} m, 3.30 N). Find the slope and the intercept, then the force predicted for an extension of 5.0 cm. Write each step number beside your work.

Work:

Answer: _____

Slide 6 of 8 · Find Coulomb’s constant from a field graph

Here k is _____, not a spring constant. $E = kQ/r^2$. E is the field strength (N/C). Q is the source charge (C). r is the distance (m). Q is _____ (horizontal). E is _____ (vertical). r and k are held _____.

Match $E = kQ/r^2$ to $y = mx$. The slope is _____. So k = _____. Multiply by r^2, do not divide.

Keep the powers of ten. Convert first: 1 μC = _____ C. Unit check: $(N/C^2)(m^2)$ = _____.

Compare with the data-sheet _____. A numerical response uses three digits in the form _____.

Example. At r = 0.500 m, J = (2.0 μC, 1.0×10^5 N/C) and K = (10.0 μC, 4.0×10^5 N/C) lie on the line of E against Q. Find k from the graph. The data sheet gives 8.99×10^9 $N \cdot m^2/C^2$.

Work:

Answer: _____

Try it. At r = 0.300 m, a line of best fit for E against Q passes through M = (3.0×10^{-6} C, 3.2×10^5 N/C) and N = (9.0×10^{-6} C, 9.5×10^5 N/C). Find the experimental value of Coulomb’s constant. Record it as $a.bc \times 10^n$ $N \cdot m^2/C^2$.

Work:

Answer: _____

Slide 7 of 8 · Straighten a curve: plot against $1/r^2$

Fixed charges $Q_1 = 3.00\ \mu C$ and $Q_2 = 2.00\ \mu C$. Use sizes only, no minus sign. F against r is a _____. A curve has no single slope: rise over run _____ with every pair of points.

Coulomb's law: $F =$ _____ $\times (1/r^2)$. F is proportional to _____, not to r.

So plot F against _____. Then $F = (kQ_1Q_2)(1/r^2)$ matches $y = mx$: a _____ with slope _____. For r = 0.500 m: $0.500^2 = 0.250$ and $1 \div 0.250 =$ _____.

The charges are in the slope, so _____ the slope by Q_1Q_2 to get k. The slope unit is $N \div m^{-2} =$ _____. Add the unit _____ yourself.

Example. $Q_1 = 3.00\ \mu C$ and $Q_2 = 2.00\ \mu C$. Points C = (2.0 m^{-2}, 0.108 N) and D = (10.0 m^{-2}, 0.524 N) lie on the line of F against $1/r^2$. Find k.

Work:

Answer: _____

Try it. Two equal charges, $Q_1 = Q_2 = 4.00\ \mu C$. A line of F against $1/r^2$ passes through G = (2.0 m^{-2}, 0.31 N) and H = (8.0 m^{-2}, 1.20 N). Find the experimental value of Coulomb's constant.

Work:

Answer: _____

Sketch (draw and label the diagram):

Slide 8 of 8 · Method: how to straighten a curve

Step 1: name the _____. Force is proportional to $1/r^2$, not to r. Step 2: plot against the quantity the law is _____: _____.

Step 3: make the new _____: square each r, then flip it. Keep an extra digit until the column is done.

Step 4: plot the new column on the _____ axis. Draw one line of best fit.

Step 5: read the slope. It holds every constant, so _____ by the ones you know. Check: the points fall on a line near the _____, and the slope unit is the vertical unit ÷ m^{-2}.

Example. Q_1 = 3.00 µC and Q_2 = 2.00 µC, with separations 1.00, 0.707, 0.500 and 0.316 m. Points C = (2.0 m^{-2}, 0.108 N) and D = (10.0 m^{-2}, 0.524 N) lie on the line of F against $1/r^2$. Find k. Write each step number beside your work.

Work:

Answer: _____

Try it. Q_1 = 2.00 µC and Q_2 = 6.00 µC. Points P = (4.0 m^{-2}, 0.43 N) and R = (12.0 m^{-2}, 1.29 N) lie on the line of F against $1/r^2$. Find the experimental value of Coulomb's constant. Write each step number beside your work.

Work:

Answer: _____

B5 · practice

Reading straight-line graphs | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass $m_p = 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B5-N01. A line of best fit passes through (0.5 s, 1.2 m) and (4.5 s, 5.2 m). Find its slope in m/s.

Work:

Final answer (with unit and sign): _____

Easy · B5-N02. Two charges are fixed. A student measures the force F at several separations r and wants a straight-line graph. Which pair of axes works?

A. F on the vertical axis, $1/r^2$ on the horizontal axis
B. F on the vertical axis, r on the horizontal axis
C. r on the vertical axis, F on the horizontal axis
D. F on the vertical axis, r^2 on the horizontal axis

Work:

Final answer (with unit and sign): _____

B5 · practice

Reading straight-line graphs | Questions 3-4 of 8

Easy · B5-N03. A line of best fit is drawn for scattered data points. Which description is correct?

A. It must pass through every data point.
B. It must start at the origin.
C. It joins the first point to the last point.
D. It follows the overall trend, with points above and below it.

Work:

Final answer (with unit and sign): _____

Medium · B5-N04. An experiment gives the line F = (40.0 N/m)x + 0.30 N, with data from x = 0.010 m to 0.080 m. Find the extension x that the line predicts for F = 2.70 N.

Work:

Final answer (with unit and sign): _____

B5 · practice

Reading straight-line graphs | Questions 5-6 of 8

Medium · B5-N05. A graph of electric field strength E against $1/r^2$ from a point charge is a straight line through the origin with slope 5390 N·m²/C. Use $k = 8.99 \times 10^9$ N·m²/C². Find the source charge in coulombs.

Work:

Final answer (with unit and sign): _____

Medium · B5-N06. A student extends the line $F = (25.0\ \text{N/m})x + 0.50\ \text{N}$ to $x = 0.30$ m, but all the measurements were made between $x = 0.02$ m and 0.12 m. Predict F and say whether you trust it.

Work:

Final answer (with unit and sign): _____

B5 · practice

Reading straight-line graphs | Questions 7-8 of 8

Challenging · B5-N07. Two charges $q_1 = 2.0 \times 10^{-6}$ C and q_2 are fixed. The force on q_2 is 5.39 N at r = 0.100 m and 0.599 N at r = 0.300 m. Plot F against $1/r^2$. Find the slope in N·m², using these two points.

Work:

Final answer (with unit and sign): _____

Challenging · B5-N08. Use your slope from the previous question (0.0539 N·m²) and k = 8.99×10^9 N·m²/C² to find q_2, given $q_1 = 2.0 \times 10^{-6}$ C. Explain what the line would show about the force if q_2 were doubled.

Work:

Final answer (with unit and sign): _____