B5 · Extra practice, worked solutions

Reading straight-line graphs | Diploma-style set of 8

Easy · B5-X01. A speed-time graph has a line of best fit through (1.0 s, 2.5 m/s) and (6.0 s, 14.0 m/s). The slope is the acceleration. Find the acceleration in m/s².

1. Givens: two points on the line, (1.0 s, 2.5 m/s) and (6.0 s, 14.0 m/s). Question: the slope, which is the acceleration.

2. Slope = $(y_2 - y_1) \div (x_2 - x_1)$.

3. = (14.0 − 2.5) ÷ (6.0 − 1.0) = 2.30 m/s².

4. The unit is the y unit over the x unit: (m/s) ÷ s = m/s².

5. Answer: 2.30 m/s²

Easy · B5-X02. A line of best fit for distance against time is d = (12 m/s)t + 4 m. What does the 4 m stand for?

A. The distance at t = 0 s
B. The speed of the object
C. The slope of the line
D. The distance at t = 12 s

1. Givens: d = (12 m/s)t + 4 m, which has the form y = mx + b. Question: what b means.

2. b is the y-intercept, the value of y when x = 0.

3. So 4 m is the distance when t = 0 s. The slope m = 12 m/s is the speed.

4. Answer: A. The distance at t = 0 s

Medium · B5-X03. A graph of field strength E against 1/r² for a point charge is a straight line through the origin. It passes through (4.00 m⁻², 216 N/C) and (25.0 m⁻², 1350 N/C). Use k = 8.99 × 10⁹ N·m²/C². Find the source charge in coulombs.

1. Givens: points (4.00 m⁻², 216 N/C) and (25.0 m⁻², 1350 N/C); k = 8.99 × 10⁹ N·m²/C². Question: Q.

2. Slope = (1350 − 216) ÷ (25.0 − 4.00) = 54.0 N·m²/C.

3. $E = kQ(1/r^2)$ matches y = mx, so the slope = kQ.

4. Q = slope ÷ k = 54.0 ÷ (8.99 × 10⁹) = 6.01 × 10⁻⁹ C.

5. Answer: 6.01 × 10⁻⁹ C

Medium · B5-X04. A cart starts from rest and speeds up steadily, so $d = \frac{1}{2}at^2$. Which pair of axes gives a straight line?

A. d on the vertical axis, t on the horizontal axis
B. t on the vertical axis, d on the horizontal axis
C. d on the vertical axis, t^2 on the horizontal axis
D. d on the vertical axis, $1/t^2$ on the horizontal axis

1. Givens: $d = \frac{1}{2}at^2$, with a constant. Question: which axes make a line.

2. Compare with $y = mx$: if $x = t^2$, then $d = (a/2)(t^2)$, a straight line through the origin.

3. The slope is $a/2$.

4. Answer: C. d on the vertical axis, t^2 on the horizontal axis

Medium · B5-X05. A spring test gives the line $F = (18.0\ \text{N/m})x + 0.40\ \text{N}$ for extensions from 0.020 m to 0.150 m. Find the extension x, in metres, that gives $F = 2.50$ N.

1. Givens: $F = (18.0\ \text{N/m})x + 0.40\ \text{N}$; $F = 2.50$ N; data from 0.020 m to 0.150 m. Question: x.

2. Solve for x: $x = (F - b) \div m$.

3. $x = (2.5 - 0.4) \div 18.0 = 0.1167$ m.

4. This is inside the measured range, so it is interpolation and is reliable.

5. Answer: 0.117 m

Challenging · B5-X06. A small charged ball sits in a uniform field. A graph of force F against test charge q is a straight line through the origin. It passes through (2.00×10^{-6} C, 3.00×10^{-3} N) and (8.00×10^{-6} C, 0.0120 N). Find the field strength in N/C.

1. Givens: points (2.00×10^{-6} C, 3.00×10^{-3} N) and (8.00×10^{-6} C, 0.0120 N). Question: the field strength E.

2. Slope = $(0.0120 - 3.00 \times 10^{-3}) \div (8.00 \times 10^{-6} - 2.00 \times 10^{-6}) = 1500$ N/C.

3. $F = qE$ matches $y = mx$, so the slope is the field strength E.

4. Answer: 1500 N/C

Challenging · B5-X07. Force measurements were made for separations from 0.10 m to 0.50 m. A student predicts the force at 0.30 m and at 0.90 m from the line of best fit. Which statement is correct?

A. Both are interpolation, so both are equally reliable.

B. 0.30 m is extrapolation; 0.90 m is interpolation.

C. Both are extrapolation, so neither can be used.

D. 0.30 m is interpolation and is reliable; 0.90 m is extrapolation and is less reliable.

1. Givens: measured range 0.10 m to 0.50 m; predictions at 0.30 m and 0.90 m. Question: which prediction is reliable.

2. 0.30 m is inside the range: interpolation.

3. 0.90 m is outside the range: extrapolation. The line may not keep going the same way, so trust it less.

4. Answer: D. 0.30 m is interpolation and is reliable; 0.90 m is extrapolation and is less reliable.

Challenging · B5-X08. A graph of force F against charge q gives the line $F = (1500\ \text{N/C})q + 2.0 \times 10^{-3}\ \text{N}$. Theory says $F = qE$, so the line should pass through the origin. Give one likely reason for the 2.0×10^{-3} N and say what you would do about it.

1. Givens: line $F = (1500\ \text{N/C})q + 2.0 \times 10^{-3}\ \text{N}$; theory $F = qE$, with zero intercept. Question: a likely reason for the intercept, and what to do.

2. The intercept is the force at $q = 0$. A force when there is no charge means something other than the electric force is acting, or the instrument reads wrongly at zero.

3. Likely reasons: the force meter was not set to zero (a zero error), or a small friction or weight force acted.

4. Fix: zero the meter before measuring, repeat the data, and use the slope (1500 N/C) for E, because the slope is not changed by a constant offset.