B5 · Reading straight-line graphs

Completed notes

Find a slope with its unit, match a graph to y = mx + b, and straighten a curve.

Slide 1 of 8 · Two variables and a line of best fit

A fair test changes one thing and measures one thing. The **manipulated** variable is the one you change. The **responding** variable is the one you measure. Everything else is held **fixed**.

The manipulated variable goes on the **horizontal** axis. The responding variable goes on the **vertical** axis. Spring test: the stretch (m) is **manipulated**. The force (N) is **responding**.

Points never fall exactly on a line. Every reading has a small **error**. A line of best fit follows the **trend**.

Read values from the **line**, never from single data points. Choose two points on the line that are **far apart**, so the error is a small share of the answer.

Example. A student changes the source charge Q and measures the electric field strength E with a probe. The probe stays 0.500 m from the source in air. Which quantity is manipulated, which is responding, which is held fixed, and which one goes on the horizontal axis?

1. Manipulated: source charge Q (horizontal axis). Responding: electric field strength E (vertical axis).

2. Fixed: the distance r = 0.500 m, and the constant k, which is a fixed number of nature.

3. If r also changed, E would depend on two things at once and the graph could not show how E depends on Q alone.

Try it. A student draws a line of best fit through six scattered points. To find the slope she reads two of the data points that are close together. Give two things she should change.

1. Read the two points from the line of best fit, not from single data points, because each data point carries its own reading error and the line averages the errors out.

2. Choose two points on the line that are far apart, so the same reading error is a small share of the rise and the run.

Slide 2 of 8 · Find the slope from the line

Slope = **rise ÷ run**. Rise is how far the line goes **up**. Run is how far it goes **across**. Take B minus A for both, in the **same order**. F_B is the force at B. x_A is the extension at A.

Slope = **$(y_2 - y_1) \div (x_2 - x_1)$**. It shows how much the vertical quantity changes for each **one unit** of the horizontal quantity.

The slope unit is the **vertical unit** ÷ the **horizontal unit**. Here F is in N and x is in m, so the slope is in **N/m**. A slope without its unit is incomplete.

Convert the run to **metres**. Keep the power of ten. A slope of 250 or 0.25 instead of 25.0 N/m means the **power of ten** was lost.

Example. Points A (2.0×10^{-2} m, 1.00 N) and B (10.0×10^{-2} m, 3.00 N) lie on the line of best fit of spring force F against extension x. Find the slope.

1. Rise = F_B − F_A (force at B minus force at A) = 3.00 N − 1.00 N = 2.00 N.

2. Run = x_B − x_A = $(10.0 - 2.0) \times 10^{-2}$ m = 8.0×10^{-2} m = 0.080 m.

3. Slope = 2.00 N ÷ 0.080 m = 25.0 N/m.

Try it. A different spring’s line of best fit passes through C = (3.0×10^{-2} m, 1.40 N) and D = (9.0×10^{-2} m, 3.80 N). Find the slope, including its unit. Show the rise, the run and the unit check.

1. Rise = F_D − F_C = 3.80 N − 1.40 N = 2.40 N.

2. Run = $(9.0 - 3.0) \times 10^{-2}$ m = 6.0×10^{-2} m = 0.060 m.

3. Slope = 2.40 N ÷ 0.060 m = 40.0 N/m. Unit check: newtons per metre, vertical over horizontal.

Slide 3 of 8 · Match the physics equation to y = mx + b

A straight line follows y = **mx + b**. m is the **slope**. b is the **vertical intercept**, where the line crosses the vertical axis (x = **0**). Hooke's law is F = kx. F is the force (N). x is the extension (m). k is the spring constant (N/m).

Match the symbols one at a time. The vertical-axis quantity plays **y**. The horizontal-axis quantity plays **x**. The number multiplying x is the **slope**. The term that stands alone is the **intercept**.

In F = kx, F plays y and x plays x. So the slope is the **spring constant k**. The law has **no** b term, so a 0.50 N intercept is not part of the law.

Sometimes the intercept is real physics. In $v = v_0 + at$, a is the constant acceleration (m/s^2) and v_0 is the starting speed (m/s). The term with no t is the **initial speed v_0**. Always ask if the physics equation has a **b term**.

Example. A cart's speed v is plotted against time t, and the line of best fit has the equation $v = v_0 + at$, where a is a constant acceleration. Using y = mx + b, what does the slope represent and what does the vertical intercept represent?

1. Match v to y and t to x. The number multiplying t is a, so the slope is the acceleration a (in m/s per s, which is m/s^2).

2. The term with no t is v_0, which plays the role of b, so the vertical intercept is the initial speed v_0.

3. Here the intercept is real physics, not an error.

Try it. A graph of the electric field strength E against the source charge Q, measured at a fixed distance r from a point source, matches $E = kQ/r^2$. Using y = mx + b, what does the slope equal, and what should the intercept be?

1. Match E to y and Q to x. The number multiplying Q is k/r^2, so the slope equals k/r^2 (r is fixed).

2. The equation $E = kQ/r^2$ has no b term, so the ideal line passes through the origin and the intercept should be 0.

Slide 4 of 8 · Predict with the whole line, intercept included

Predict with the whole line: y = **mx + b**, intercept included. The spring line is F = (25.0 N/m)x + 0.50 N.

To find x from F, write the equation with F in it. **Subtract the intercept** first. Then **divide by the slope**. For F = 1.20 N: 0.70 = 25.0x, so x = **0.0280 m**. F = kx alone gives 0.0480 m and leaves out the 0.50 N.

The line is in SI units, so convert first: 6.0 cm = **0.060** m. Forgetting to convert is a classic wrong choice.

Inside the measured range is **interpolation**. It is reliable. Past the range is **extrapolation**. It is less reliable, because the line may not stay straight.

Example. The spring line is F = (25.0 N/m)x + 0.50 N, from measured extensions up to 12 × 10^{-2} m. The extension is 6.0 cm. Find the predicted force.

1. Convert: 6.0 cm = 0.060 m.

2. F = (25.0 N/m)(0.060 m) + 0.50 N = 1.50 N + 0.50 N = 2.00 N.

3. Reading the graph at 6.0 × 10^{-2} m gives the same 2.00 N.

Try it. The same line is F = (25.0 N/m)x + 0.50 N. A force of 2.30 N is applied. Find the extension this experiment predicts, in metres.

1. 2.30 N = (25.0 N/m)x + 0.50 N.

2. Subtract the intercept: 1.80 N = (25.0 N/m)x.

3. x = 1.80 N ÷ 25.0 N/m = 0.072 m.

4. Ignoring the intercept would give 0.092 m, which is too large by 0.020 m.

Slide 5 of 8 · Method: how to get slope and constants from a graph

Step 1: pick **two points** on the line. Use grid crossings, far apart. Use line points, not **data points**.

Step 2: **slope** = rise ÷ run. Keep the power of ten. The unit is the vertical unit ÷ the **horizontal unit**.

Step 3: match y = mx + b. Step 4: say what the slope and the **intercept** mean, with units. Does the physics equation have a b term?

Step 5: predict with the **whole line**. Check: the unit and the size fit the picture.

Example. The spring line is F = (25.0 N/m)x + 0.50 N. A and B lie on it at (2.0×10^{-2} m, 1.00 N) and (10.0×10^{-2} m, 3.00 N). Find the spring constant, then the extension for a force of 1.20 N. Write each step number beside your work.

1. Step 1: A (2.0×10^{-2} m, 1.00 N) and B (10.0×10^{-2} m, 3.00 N), both on the line.

2. Step 2: rise = 3.00 − 1.00 = 2.00 N. Run = $(10.0 - 2.0) \times 10^{-2}$ m = 0.080 m. Slope = 2.00 ÷ 0.080 = 25.0 N/m.

3. Steps 3 and 4: F plays y and x plays x, so 25.0 N/m is the spring constant. The intercept, 0.50 N, is not in Hooke’s law: it has no b.

4. Step 5: 1.20 = 25.0x + 0.50, so 0.70 = 25.0x and x = 0.0280 m.

Try it. A line of best fit for another spring passes through (2.0×10^{-2} m, 1.50 N) and (8.0×10^{-2} m, 3.30 N). Find the slope and the intercept, then the force predicted for an extension of 5.0 cm. Write each step number beside your work.

1. Step 2: rise = 3.30 − 1.50 = 1.80 N. Run = $(8.0 - 2.0) \times 10^{-2}$ m = 0.060 m. Slope = 1.80 ÷ 0.060 = 30.0 N/m.

2. Steps 3 and 4: F = 30.0x + b. Put in the first point: 1.50 = 30.0(0.020) + b, so b = 0.90 N.

3. Step 5: 5.0 cm = 0.050 m. F = 30.0(0.050) + 0.90 = 1.50 + 0.90 = 2.40 N.

Slide 6 of 8 · Find Coulomb’s constant from a field graph

Here k is **Coulomb’s constant**, not a spring constant. $E = kQ/r^2$. E is the field strength (N/C). Q is the source charge (C). r is the distance (m). Q is **manipulated** (horizontal). E is **responding** (vertical). r and k are held **fixed**.

Match $E = kQ/r^2$ to $y = mx$. The slope is $\mathbf{k/r^2}$. So k = **slope × r^2**. Multiply by r^2, do not divide.

Keep the powers of ten. Convert first: 1 μC = $\mathbf{1 \times 10^{-6}}$ C. Unit check: $(N/C^2)(m^2) = \mathbf{N \cdot m^2/C^2}$.

Compare with the data-sheet $\mathbf{8.99 \times 10^9\ N \cdot m^2/C^2}$. A numerical response uses three digits in the form $\mathbf{a.bc \times 10^n}$.

Example. At r = 0.500 m, J = (2.0 μC, 1.0×10^5 N/C) and K = (10.0 μC, 4.0×10^5 N/C) lie on the line of E against Q. Find k from the graph. The data sheet gives 8.99×10^9 $N \cdot m^2/C^2$.

1. Rise = $(4.0 - 1.0) \times 10^5$ N/C = 3.0×10^5 N/C. Run = $(10.0 - 2.0) \times 10^{-6}$ C = 8.0×10^{-6} C.

2. Slope = 3.75×10^{10} N/C^2.

3. $k = \text{slope} \times r^2 = (3.75 \times 10^{10})(0.500)^2 = 9.38 \times 10^9$ $N \cdot m^2/C^2$.

4. Unit check: $(N/C^2)(m^2) = N \cdot m^2/C^2$. It differs from 8.99×10^9 by about 4 %, close enough for a classroom experiment.

Try it. At r = 0.300 m, a line of best fit for E against Q passes through M = (3.0×10^{-6} C, 3.2×10^5 N/C) and N = (9.0×10^{-6} C, 9.5×10^5 N/C). Find the experimental value of Coulomb’s constant. Record it as $a.bc \times 10^n$ $N \cdot m^2/C^2$.

1. Rise = $(9.5 - 3.2) \times 10^5$ N/C = 6.3×10^5 N/C. Run = $(9.0 - 3.0) \times 10^{-6}$ C = 6.0×10^{-6} C.

2. Slope = 1.05×10^{11} N/C^2.

3. $k = \text{slope} \times r^2 = (1.05 \times 10^{11}\ N/C^2)(0.300\ m)^2 = 9.45 \times 10^9$ $N \cdot m^2/C^2$.

4. Record 9.45×10^9.

Slide 7 of 8 · Straighten a curve: plot against $1/r^2$

Fixed charges $Q_1 = 3.00\ \mu C$ and $Q_2 = 2.00\ \mu C$. Use sizes only, no minus sign. F against r is a **curve**. A curve has no single slope: rise over run **changes** with every pair of points.

Coulomb's law: $F = \mathbf{kQ_1Q_2} \times (1/r^2)$. F is proportional to $\mathbf{1/r^2}$, not to r.

So plot F against $\mathbf{1/r^2}$. Then $F = (kQ_1Q_2)(1/r^2)$ matches $y = mx$: a **straight line** with slope $\mathbf{kQ_1Q_2}$. For r = 0.500 m: $0.500^2 = 0.250$ and $1 \div 0.250 = \mathbf{4.00\ m^{-2}}$.

The charges are in the slope, so **divide** the slope by Q_1Q_2 to get k. The slope unit is N ÷ m^{-2} = $\mathbf{N \cdot m^2}$. Add the unit $\mathbf{N \cdot m^2/C^2}$ yourself.

Example. $Q_1 = 3.00\ \mu C$ and $Q_2 = 2.00\ \mu C$. Points C = (2.0 m^{-2}, 0.108 N) and D = (10.0 m^{-2}, 0.524 N) lie on the line of F against $1/r^2$. Find k.

1. Slope = (0.524 − 0.108) N ÷ (10.0 − 2.0) m^{-2} = 0.0520 N·m^2.

2. Slope = kQ_1Q_2, the charge product is $(3.00 \times 10^{-6})(2.00 \times 10^{-6}) = 6.00 \times 10^{-12}\ C^2$, so $k = 0.0520 \div (6.00 \times 10^{-12})$.

3. $k = 8.67 \times 10^9\ N \cdot m^2/C^2$.

Try it. Two equal charges, $Q_1 = Q_2 = 4.00\ \mu C$. A line of F against $1/r^2$ passes through G = (2.0 m^{-2}, 0.31 N) and H = (8.0 m^{-2}, 1.20 N). Find the experimental value of Coulomb's constant.

1. Slope = (1.20 − 0.31) N ÷ (8.0 − 2.0) m^{-2} = 0.890 ÷ 6.0 = 0.1483 N·m^2.

2. Charge product = $(4.00 \times 10^{-6}\ C)^2 = 1.60 \times 10^{-11}\ C^2$.

3. k = slope ÷ $(Q_1Q_2) = 0.1483 \div (1.60 \times 10^{-11}) = 9.27 \times 10^9\ N \cdot m^2/C^2$.

Slide 8 of 8 · Method: how to straighten a curve

Step 1: name the **relationship**. Force is proportional to $1/r^2$, not to r. Step 2: plot against the quantity the law is **proportional to**: **$1/r^2$**.

Step 3: make the new **column**: square each r, then flip it. Keep an extra digit until the column is done.

Step 4: plot the new column on the **horizontal** axis. Draw one line of best fit.

Step 5: read the slope. It holds every constant, so **divide** by the ones you know. Check: the points fall on a line near the **origin**, and the slope unit is the vertical unit ÷ m^{-2}.

Example. $Q_1 = 3.00\ \mu C$ and $Q_2 = 2.00\ \mu C$, with separations 1.00, 0.707, 0.500 and 0.316 m. Points C = (2.0 m^{-2}, 0.108 N) and D = (10.0 m^{-2}, 0.524 N) lie on the line of F against $1/r^2$. Find k. Write each step number beside your work.

1. Step 1: $F = kQ_1Q_2 \times (1/r^2)$, so F is proportional to $1/r^2$. Step 2: plot F against $1/r^2$.

2. Step 3: $0.500^2 = 0.250$ and $1 \div 0.250 = 4.00$. The column is 1.00, 2.00, 4.00 and 10.0 m^{-2}.

3. Steps 4 and 5: slope = $(0.524 - 0.108) \div (10.0 - 2.0) = 0.0520\ N{\cdot}m^2$.

4. The charge product is $(3.00 \times 10^{-6})(2.00 \times 10^{-6}) = 6.00 \times 10^{-12}\ C^2$, so $k = 0.0520 \div (6.00 \times 10^{-12}) = 8.67 \times 10^9\ N{\cdot}m^2/C^2$.

Try it. $Q_1 = 2.00\ \mu C$ and $Q_2 = 6.00\ \mu C$. Points P = (4.0 m^{-2}, 0.43 N) and R = (12.0 m^{-2}, 1.29 N) lie on the line of F against $1/r^2$. Find the experimental value of Coulomb's constant. Write each step number beside your work.

1. Steps 1 and 2: F is proportional to $1/r^2$, so the line is F against $1/r^2$ and the slope is kQ_1Q_2.

2. Step 5: slope = $(1.29 - 0.43) \div (12.0 - 4.0) = 0.86 \div 8.0 = 0.1075\ N{\cdot}m^2$.

3. The charge product is $(2.00 \times 10^{-6})(6.00 \times 10^{-6}) = 1.20 \times 10^{-11}\ C^2$.

4. $k = 0.1075 \div (1.20 \times 10^{-11}) = 8.96 \times 10^9\ N{\cdot}m^2/C^2$.

B5 · Worked practice

Reading straight-line graphs | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass $m_p = 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B5-N01. A line of best fit passes through (0.5 s, 1.2 m) and (4.5 s, 5.2 m). Find its slope in m/s.

1. Givens: two points on the line, (0.50 s, 1.20 m) and (4.50 s, 5.20 m). Question: the slope.

2. Slope = $(y_2 - y_1) \div (x_2 - x_1)$.

3. = $(5.2 - 1.2) \div (4.5 - 0.5) = 1.00$ m/s.

4. Answer: 1.00 m/s

Easy · B5-N02. Two charges are fixed. A student measures the force F at several separations r and wants a straight-line graph. Which pair of axes works?

A. F on the vertical axis, $1/r^2$ on the horizontal axis
B. F on the vertical axis, r on the horizontal axis
C. r on the vertical axis, F on the horizontal axis
D. F on the vertical axis, r^2 on the horizontal axis

1. Givens: $F = kq_1q_2/r^2$ with the charges fixed. Question: which axes give a line.

2. F is proportional to $1/r^2$. Plotting F against $1/r^2$ follows $y = mx$ with $m = kq_1q_2$.

3. F against r is a curve. F against r^2 is also a curve, because F gets smaller as r^2 gets larger.

4. Answer: A. F on the vertical axis, $1/r^2$ on the horizontal axis

B5 · Worked practice

Reading straight-line graphs | Questions 3-4 of 8

Easy · B5-N03. A line of best fit is drawn for scattered data points. Which description is correct?

A. It must pass through every data point.
B. It must start at the origin.
C. It joins the first point to the last point.
D. It follows the overall trend, with points above and below it.

1. Givens: scattered measurements. Question: what a line of best fit is.

2. The line shows the trend. Measurement error puts points on both sides, so some points sit above and some below.

3. Answer: D. It follows the overall trend, with points above and below it.

Medium · B5-N04. An experiment gives the line F = (40.0 N/m)x + 0.30 N, with data from x = 0.010 m to 0.080 m. Find the extension x that the line predicts for F = 2.70 N.

1. Givens: F = (40.0 N/m)x + 0.30 N; F = 2.70 N. Question: x.

2. Solve for x: x = (F − b) ÷ m.

3. x = (2.7 − 0.3) ÷ 40.0 = 0.0600 m.

4. This is inside the measured range (0.010–0.080 m), so it is interpolation and is reliable.

5. Answer: 0.0600 m

B5 · Worked practice

Reading straight-line graphs | Questions 5-6 of 8

Medium · B5-N05. A graph of electric field strength E against $1/r^2$ from a point charge is a straight line through the origin with slope 5390 $N \cdot m^2/C$. Use $k = 8.99 \times 10^9$ $N \cdot m^2/C^2$. Find the source charge in coulombs.

1. Givens: slope = 5390 $N \cdot m^2/C$; $k = 8.99 \times 10^9$ $N \cdot m^2/C^2$. Question: Q.

2. $E = kQ(1/r^2)$ matches $y = mx$, so the slope = kQ.

3. $Q = \text{slope} \div k = 5390 \div (8.99 \times 10^9) = 6.00 \times 10^{-7}$ C.

4. Answer: 6.00×10^{-7} C

Medium · B5-N06. A student extends the line $F = (25.0\ N/m)x + 0.50\ N$ to x = 0.30 m, but all the measurements were made between x = 0.02 m and 0.12 m. Predict F and say whether you trust it.

1. Givens: $F = (25.0\ N/m)x + 0.50\ N$; data only from 0.02 m to 0.12 m. Question: F at 0.30 m and how reliable it is.

2. $F = 25.0(0.30) + 0.50 = 8.0$ N.

3. 0.30 m is far outside the measured range, so this is extrapolation. The spring may stop following a straight line (it could stretch permanently), so the prediction is not reliable.

B5 · Worked practice

Reading straight-line graphs | Questions 7-8 of 8

Challenging · B5-N07. Two charges $q_1 = 2.0 \times 10^{-6}$ C and q_2 are fixed. The force on q_2 is 5.39 N at r = 0.100 m and 0.599 N at r = 0.300 m. Plot F against $1/r^2$. Find the slope in $N \cdot m^2$, using these two points.

1. Givens: F = 5.39 N at r = 0.100 m; F = 0.599 N at r = 0.300 m. Question: the slope of F against $1/r^2$.

2. Convert: $1/r^2 = 1 \div 0.100^2 = 100\ m^{-2}$ and $1 \div 0.300^2 = 11.1\ m^{-2}$.

3. Slope = $(5.39 - 0.599) \div (100 - 11.1) = 0.0539\ N \cdot m^2$ (about 0.054).

4. The unit is $N \div m^{-2} = N \cdot m^2$.

5. Answer: $0.0539\ N \cdot m^2$

Challenging · B5-N08. Use your slope from the previous question ($0.0539\ N \cdot m^2$) and $k = 8.99 \times 10^9\ N \cdot m^2/C^2$ to find q_2, given $q_1 = 2.0 \times 10^{-6}$ C. Explain what the line would show about the force if q_2 were doubled.

1. Givens: slope = $kq_1q_2 = 0.0539\ N \cdot m^2$; $q_1 = 2.0 \times 10^{-6}$ C. Question: q_2, and the effect of doubling q_2.

2. q_2 = slope ÷ $(kq_1) = 0.0539 \div ((8.99 \times 10^9)(2.0 \times 10^{-6})) = 3.00 \times 10^{-6}$ C.

3. If q_2 doubled, the slope kq_1q_2 would double, so the line would be twice as steep.