B5 · Reading straight-line graphs

Completed notes

Find a slope with its unit, match a graph to y = mx + b, and straighten a curve.

Slide 1 of 8 · Two variables and a line of best fit

A fair test changes one thing and measures one thing. The **manipulated** variable is the one you change. The **responding** variable is the one you measure. Everything else is held **fixed**.

The manipulated variable goes on the **horizontal** axis. The responding variable goes on the **vertical** axis. Spring test: the stretch (m) is **manipulated**. The force (N) is **responding**.

Points never fall exactly on a line. Every reading has a small **error**. A line of best fit follows the **trend**.

Read values from the **line**, never from single data points. Choose two points on the line that are **far apart**, so the error is a small share of the answer.

Example. A student changes the source charge Q and measures the electric field strength E with a probe. The probe stays 0.500 m from the source in air. Which quantity is manipulated, which is responding, which is held fixed, and which one goes on the horizontal axis?

1. Manipulated: source charge Q (horizontal axis). Responding: electric field strength E (vertical axis).

2. Fixed: the distance r = 0.500 m, and the constant k, which is a fixed number of nature.

3. If r also changed, E would depend on two things at once and the graph could not show how E depends on Q alone.

Try it. A student draws a line of best fit through six scattered points. To find the slope she reads two of the data points that are close together. Give two things she should change.

1. Read the two points from the line of best fit, not from single data points, because each data point carries its own reading error and the line averages the errors out.

2. Choose two points on the line that are far apart, so the same reading error is a small share of the rise and the run.

Slide 2 of 8 · Find the slope from the line

Slope = **rise ÷ run**. Rise is how far the line goes **up**. Run is how far it goes **across**. Take B minus A for both, in the **same order**. F_B is the force at B. x_A is the extension at A.

Slope = **$(y_2 - y_1) \div (x_2 - x_1)$**. It shows how much the vertical quantity changes for each **one unit** of the horizontal quantity.

The slope unit is the **vertical unit** ÷ the **horizontal unit**. Here F is in N and x is in m, so the slope is in **N/m**. A slope without its unit is incomplete.

Convert the run to **metres**. Keep the power of ten. A slope of 250 or 0.25 instead of 25.0 N/m means the **power of ten** was lost.

Example. Points A (2.0×10^{-2} m, 1.00 N) and B (10.0×10^{-2} m, 3.00 N) lie on the line of best fit of spring force F against extension x. Find the slope.

1. Rise = F_B − F_A (force at B minus force at A) = 3.00 N − 1.00 N = 2.00 N.

2. Run = x_B − x_A = $(10.0 - 2.0) \times 10^{-2}$ m = 8.0×10^{-2} m = 0.080 m.

3. Slope = 2.00 N ÷ 0.080 m = 25.0 N/m.

Try it. A different spring’s line of best fit passes through C = (3.0×10^{-2} m, 1.40 N) and D = (9.0×10^{-2} m, 3.80 N). Find the slope, including its unit. Show the rise, the run and the unit check.

1. Rise = F_D − F_C = 3.80 N − 1.40 N = 2.40 N.

2. Run = $(9.0 - 3.0) \times 10^{-2}$ m = 6.0×10^{-2} m = 0.060 m.

3. Slope = 2.40 N ÷ 0.060 m = 40.0 N/m. Unit check: newtons per metre, vertical over horizontal.

Slide 3 of 8 · Match the physics equation to y = mx + b

A straight line follows y = **mx + b**. m is the **slope**. b is the **vertical intercept**, where the line crosses the vertical axis (x = **0**). Hooke's law is F = kx. F is the force (N). x is the extension (m). k is the spring constant (N/m).

Match the symbols one at a time. The vertical-axis quantity plays **y**. The horizontal-axis quantity plays **x**. The number multiplying x is the **slope**. The term that stands alone is the **intercept**.

In F = kx, F plays y and x plays x. So the slope is the **spring constant k**. The law has **no** b term, so a 0.50 N intercept is not part of the law.

Sometimes the intercept is real physics. In $v = v_0 + at$, a is the constant acceleration (m/s^2) and v_0 is the starting speed (m/s). The term with no t is the **initial speed v_0**. Always ask if the physics equation has a **b term**.

Example. A cart's speed v is plotted against time t, and the line of best fit has the equation $v = v_0 + at$, where a is a constant acceleration. Using y = mx + b, what does the slope represent and what does the vertical intercept represent?

1. Match v to y and t to x. The number multiplying t is a, so the slope is the acceleration a (in m/s per s, which is m/s^2).

2. The term with no t is v_0, which plays the role of b, so the vertical intercept is the initial speed v_0.

3. Here the intercept is real physics, not an error.

Try it. A graph of the electric field strength E against the source charge Q, measured at a fixed distance r from a point source, matches $E = kQ/r^2$. Using y = mx + b, what does the slope equal, and what should the intercept be?

1. Match E to y and Q to x. The number multiplying Q is k/r^2, so the slope equals k/r^2 (r is fixed).

2. The equation $E = kQ/r^2$ has no b term, so the ideal line passes through the origin and the intercept should be 0.

Slide 4 of 8 · Predict with the whole line, intercept included

Predict with the whole line: y = **mx + b**, intercept included. The spring line is F = (25.0 N/m)x + 0.50 N.

To find x from F, write the equation with F in it. **Subtract the intercept** first. Then **divide by the slope**. For F = 1.20 N: 0.70 = 25.0x, so x = **0.0280 m**. F = kx alone gives 0.0480 m and leaves out the 0.50 N.

The line is in SI units, so convert first: 6.0 cm = **0.060** m. Forgetting to convert is a classic wrong choice.

Inside the measured range is **interpolation**. It is reliable. Past the range is **extrapolation**. It is less reliable, because the line may not stay straight.

Example. The spring line is F = (25.0 N/m)x + 0.50 N, from measured extensions up to 12 × 10^{-2} m. The extension is 6.0 cm. Find the predicted force.

1. Convert: 6.0 cm = 0.060 m.

2. F = (25.0 N/m)(0.060 m) + 0.50 N = 1.50 N + 0.50 N = 2.00 N.

3. Reading the graph at 6.0 × 10^{-2} m gives the same 2.00 N.

Try it. The same line is F = (25.0 N/m)x + 0.50 N. A force of 2.30 N is applied. Find the extension this experiment predicts, in metres.

1. 2.30 N = (25.0 N/m)x + 0.50 N.

2. Subtract the intercept: 1.80 N = (25.0 N/m)x.

3. x = 1.80 N ÷ 25.0 N/m = 0.072 m.

4. Ignoring the intercept would give 0.092 m, which is too large by 0.020 m.

Slide 5 of 8 · Method: how to get slope and constants from a graph

Step 1: pick **two points** on the line. Use grid crossings, far apart. Use line points, not **data points**.

Step 2: **slope** = rise ÷ run. Keep the power of ten. The unit is the vertical unit ÷ the **horizontal unit**.

Step 3: match y = mx + b. Step 4: say what the slope and the **intercept** mean, with units. Does the physics equation have a b term?

Step 5: predict with the **whole line**. Check: the unit and the size fit the picture.

Example. The spring line is F = (25.0 N/m)x + 0.50 N. A and B lie on it at (2.0×10^{-2} m, 1.00 N) and (10.0×10^{-2} m, 3.00 N). Find the spring constant, then the extension for a force of 1.20 N. Write each step number beside your work.

1. Step 1: A (2.0×10^{-2} m, 1.00 N) and B (10.0×10^{-2} m, 3.00 N), both on the line.

2. Step 2: rise = 3.00 − 1.00 = 2.00 N. Run = $(10.0 - 2.0) \times 10^{-2}$ m = 0.080 m. Slope = 2.00 ÷ 0.080 = 25.0 N/m.

3. Steps 3 and 4: F plays y and x plays x, so 25.0 N/m is the spring constant. The intercept, 0.50 N, is not in Hooke’s law: it has no b.

4. Step 5: 1.20 = 25.0x + 0.50, so 0.70 = 25.0x and x = 0.0280 m.

Try it. A line of best fit for another spring passes through (2.0×10^{-2} m, 1.50 N) and (8.0×10^{-2} m, 3.30 N). Find the slope and the intercept, then the force predicted for an extension of 5.0 cm. Write each step number beside your work.

1. Step 2: rise = 3.30 − 1.50 = 1.80 N. Run = $(8.0 - 2.0) \times 10^{-2}$ m = 0.060 m. Slope = 1.80 ÷ 0.060 = 30.0 N/m.

2. Steps 3 and 4: F = 30.0x + b. Put in the first point: 1.50 = 30.0(0.020) + b, so b = 0.90 N.

3. Step 5: 5.0 cm = 0.050 m. F = 30.0(0.050) + 0.90 = 1.50 + 0.90 = 2.40 N.

Slide 6 of 8 · Find Coulomb’s constant from a field graph

Here k is **Coulomb’s constant**, not a spring constant. $E = kQ/r^2$. E is the field strength (N/C). Q is the source charge (C). r is the distance (m). Q is **manipulated** (horizontal). E is **responding** (vertical). r and k are held **fixed**.

Match $E = kQ/r^2$ to $y = mx$. The slope is $\mathbf{k/r^2}$. So k = **slope × r^2**. Multiply by r^2, do not divide.

Keep the powers of ten. Convert first: 1 μC = $\mathbf{1 \times 10^{-6}}$ C. Unit check: $(N/C^2)(m^2) = \mathbf{N \cdot m^2/C^2}$.

Compare with the data-sheet $\mathbf{8.99 \times 10^9\ N \cdot m^2/C^2}$. A numerical response uses three digits in the form $\mathbf{a.bc \times 10^n}$.

Example. At r = 0.500 m, J = (2.0 μC, 1.0×10^5 N/C) and K = (10.0 μC, 4.0×10^5 N/C) lie on the line of E against Q. Find k from the graph. The data sheet gives 8.99×10^9 $N \cdot m^2/C^2$.

1. Rise = $(4.0 - 1.0) \times 10^5$ N/C = 3.0×10^5 N/C. Run = $(10.0 - 2.0) \times 10^{-6}$ C = 8.0×10^{-6} C.

2. Slope = 3.75×10^{10} N/C^2.

3. $k = \text{slope} \times r^2 = (3.75 \times 10^{10})(0.500)^2 = 9.38 \times 10^9$ $N \cdot m^2/C^2$.

4. Unit check: $(N/C^2)(m^2) = N \cdot m^2/C^2$. It differs from 8.99×10^9 by about 4 %, close enough for a classroom experiment.

Try it. At r = 0.300 m, a line of best fit for E against Q passes through M = (3.0×10^{-6} C, 3.2×10^5 N/C) and N = (9.0×10^{-6} C, 9.5×10^5 N/C). Find the experimental value of Coulomb’s constant. Record it as $a.bc \times 10^n$ $N \cdot m^2/C^2$.

1. Rise = $(9.5 - 3.2) \times 10^5$ N/C = 6.3×10^5 N/C. Run = $(9.0 - 3.0) \times 10^{-6}$ C = 6.0×10^{-6} C.

2. Slope = 1.05×10^{11} N/C^2.

3. $k = \text{slope} \times r^2 = (1.05 \times 10^{11}\ N/C^2)(0.300\ m)^2 = 9.45 \times 10^9$ $N \cdot m^2/C^2$.

4. Record 9.45×10^9.

Slide 7 of 8 · Straighten a curve: plot against $1/r^2$

Fixed charges $Q_1 = 3.00\ \mu C$ and $Q_2 = 2.00\ \mu C$. Use sizes only, no minus sign. F against r is a **curve**. A curve has no single slope: rise over run **changes** with every pair of points.

Coulomb's law: $F = \mathbf{kQ_1Q_2} \times (1/r^2)$. F is proportional to $\mathbf{1/r^2}$, not to r.

So plot F against $\mathbf{1/r^2}$. Then $F = (kQ_1Q_2)(1/r^2)$ matches $y = mx$: a **straight line** with slope $\mathbf{kQ_1Q_2}$. For r = 0.500 m: $0.500^2 = 0.250$ and $1 \div 0.250 = \mathbf{4.00\ m^{-2}}$.

The charges are in the slope, so **divide** the slope by Q_1Q_2 to get k. The slope unit is N ÷ m^{-2} = $\mathbf{N \cdot m^2}$. Add the unit $\mathbf{N \cdot m^2/C^2}$ yourself.

Example. $Q_1 = 3.00\ \mu C$ and $Q_2 = 2.00\ \mu C$. Points C = (2.0 m^{-2}, 0.108 N) and D = (10.0 m^{-2}, 0.524 N) lie on the line of F against $1/r^2$. Find k.

1. Slope = (0.524 − 0.108) N ÷ (10.0 − 2.0) m^{-2} = 0.0520 N·m^2.

2. Slope = kQ_1Q_2, the charge product is $(3.00 \times 10^{-6})(2.00 \times 10^{-6}) = 6.00 \times 10^{-12}\ C^2$, so $k = 0.0520 \div (6.00 \times 10^{-12})$.

3. $k = 8.67 \times 10^9\ N \cdot m^2/C^2$.

Try it. Two equal charges, $Q_1 = Q_2 = 4.00\ \mu C$. A line of F against $1/r^2$ passes through G = (2.0 m^{-2}, 0.31 N) and H = (8.0 m^{-2}, 1.20 N). Find the experimental value of Coulomb's constant.

1. Slope = (1.20 − 0.31) N ÷ (8.0 − 2.0) m^{-2} = 0.890 ÷ 6.0 = 0.1483 N·m^2.

2. Charge product = $(4.00 \times 10^{-6}\ C)^2 = 1.60 \times 10^{-11}\ C^2$.

3. k = slope ÷ $(Q_1Q_2) = 0.1483 \div (1.60 \times 10^{-11}) = 9.27 \times 10^9\ N \cdot m^2/C^2$.

Slide 8 of 8 · Method: how to straighten a curve

Step 1: name the **relationship**. Force is proportional to $1/r^2$, not to r. Step 2: plot against the quantity the law is **proportional to**: **$1/r^2$**.

Step 3: make the new **column**: square each r, then flip it. Keep an extra digit until the column is done.

Step 4: plot the new column on the **horizontal** axis. Draw one line of best fit.

Step 5: read the slope. It holds every constant, so **divide** by the ones you know. Check: the points fall on a line near the **origin**, and the slope unit is the vertical unit ÷ m^{-2}.

Example. $Q_1 = 3.00\ \mu C$ and $Q_2 = 2.00\ \mu C$, with separations 1.00, 0.707, 0.500 and 0.316 m. Points C = (2.0 m^{-2}, 0.108 N) and D = (10.0 m^{-2}, 0.524 N) lie on the line of F against $1/r^2$. Find k. Write each step number beside your work.

1. Step 1: $F = kQ_1Q_2 \times (1/r^2)$, so F is proportional to $1/r^2$. Step 2: plot F against $1/r^2$.

2. Step 3: $0.500^2 = 0.250$ and $1 \div 0.250 = 4.00$. The column is 1.00, 2.00, 4.00 and 10.0 m^{-2}.

3. Steps 4 and 5: slope = $(0.524 - 0.108) \div (10.0 - 2.0) = 0.0520\ N{\cdot}m^2$.

4. The charge product is $(3.00 \times 10^{-6})(2.00 \times 10^{-6}) = 6.00 \times 10^{-12}\ C^2$, so $k = 0.0520 \div (6.00 \times 10^{-12}) = 8.67 \times 10^9\ N{\cdot}m^2/C^2$.

Try it. $Q_1 = 2.00\ \mu C$ and $Q_2 = 6.00\ \mu C$. Points P = (4.0 m^{-2}, 0.43 N) and R = (12.0 m^{-2}, 1.29 N) lie on the line of F against $1/r^2$. Find the experimental value of Coulomb's constant. Write each step number beside your work.

1. Steps 1 and 2: F is proportional to $1/r^2$, so the line is F against $1/r^2$ and the slope is kQ_1Q_2.

2. Step 5: slope = $(1.29 - 0.43) \div (12.0 - 4.0) = 0.86 \div 8.0 = 0.1075\ N{\cdot}m^2$.

3. The charge product is $(2.00 \times 10^{-6})(6.00 \times 10^{-6}) = 1.20 \times 10^{-11}\ C^2$.

4. $k = 0.1075 \div (1.20 \times 10^{-11}) = 8.96 \times 10^9\ N{\cdot}m^2/C^2$.

B5 · Worked practice

Reading straight-line graphs | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass $m_p = 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B5-N01. A line of best fit passes through (0.5 s, 1.2 m) and (4.5 s, 5.2 m). Find its slope in m/s.

1. Givens: two points on the line, (0.50 s, 1.20 m) and (4.50 s, 5.20 m). Question: the slope.

2. Slope = $(y_2 - y_1) \div (x_2 - x_1)$.

3. = $(5.2 - 1.2) \div (4.5 - 0.5) = 1.00$ m/s.

4. Answer: 1.00 m/s

Easy · B5-N02. Two charges are fixed. A student measures the force F at several separations r and wants a straight-line graph. Which pair of axes works?

A. F on the vertical axis, $1/r^2$ on the horizontal axis
B. F on the vertical axis, r on the horizontal axis
C. r on the vertical axis, F on the horizontal axis
D. F on the vertical axis, r^2 on the horizontal axis

1. Givens: $F = kq_1q_2/r^2$ with the charges fixed. Question: which axes give a line.

2. F is proportional to $1/r^2$. Plotting F against $1/r^2$ follows $y = mx$ with $m = kq_1q_2$.

3. F against r is a curve. F against r^2 is also a curve, because F gets smaller as r^2 gets larger.

4. Answer: A. F on the vertical axis, $1/r^2$ on the horizontal axis

B5 · Worked practice

Reading straight-line graphs | Questions 3-4 of 8

Easy · B5-N03. A line of best fit is drawn for scattered data points. Which description is correct?

A. It must pass through every data point.
B. It must start at the origin.
C. It joins the first point to the last point.
D. It follows the overall trend, with points above and below it.

1. Givens: scattered measurements. Question: what a line of best fit is.

2. The line shows the trend. Measurement error puts points on both sides, so some points sit above and some below.

3. Answer: D. It follows the overall trend, with points above and below it.

Medium · B5-N04. An experiment gives the line F = (40.0 N/m)x + 0.30 N, with data from x = 0.010 m to 0.080 m. Find the extension x that the line predicts for F = 2.70 N.

1. Givens: F = (40.0 N/m)x + 0.30 N; F = 2.70 N. Question: x.

2. Solve for x: x = (F − b) ÷ m.

3. x = (2.7 − 0.3) ÷ 40.0 = 0.0600 m.

4. This is inside the measured range (0.010–0.080 m), so it is interpolation and is reliable.

5. Answer: 0.0600 m

B5 · Worked practice

Reading straight-line graphs | Questions 5-6 of 8

Medium · B5-N05. A graph of electric field strength E against $1/r^2$ from a point charge is a straight line through the origin with slope 5390 $N \cdot m^2/C$. Use $k = 8.99 \times 10^9$ $N \cdot m^2/C^2$. Find the source charge in coulombs.

1. Givens: slope = 5390 $N \cdot m^2/C$; $k = 8.99 \times 10^9$ $N \cdot m^2/C^2$. Question: Q.

2. $E = kQ(1/r^2)$ matches $y = mx$, so the slope = kQ.

3. $Q = \text{slope} \div k = 5390 \div (8.99 \times 10^9) = 6.00 \times 10^{-7}$ C.

4. Answer: 6.00×10^{-7} C

Medium · B5-N06. A student extends the line $F = (25.0\ N/m)x + 0.50\ N$ to x = 0.30 m, but all the measurements were made between x = 0.02 m and 0.12 m. Predict F and say whether you trust it.

1. Givens: $F = (25.0\ N/m)x + 0.50\ N$; data only from 0.02 m to 0.12 m. Question: F at 0.30 m and how reliable it is.

2. $F = 25.0(0.30) + 0.50 = 8.0$ N.

3. 0.30 m is far outside the measured range, so this is extrapolation. The spring may stop following a straight line (it could stretch permanently), so the prediction is not reliable.

B5 · Worked practice

Reading straight-line graphs | Questions 7-8 of 8

Challenging · B5-N07. Two charges $q_1 = 2.0 \times 10^{-6}$ C and q_2 are fixed. The force on q_2 is 5.39 N at r = 0.100 m and 0.599 N at r = 0.300 m. Plot F against $1/r^2$. Find the slope in $N \cdot m^2$, using these two points.

1. Givens: F = 5.39 N at r = 0.100 m; F = 0.599 N at r = 0.300 m. Question: the slope of F against $1/r^2$.

2. Convert: $1/r^2 = 1 \div 0.100^2 = 100\ m^{-2}$ and $1 \div 0.300^2 = 11.1\ m^{-2}$.

3. Slope = $(5.39 - 0.599) \div (100 - 11.1) = 0.0539\ N \cdot m^2$ (about 0.054).

4. The unit is $N \div m^{-2} = N \cdot m^2$.

5. Answer: $0.0539\ N \cdot m^2$

Challenging · B5-N08. Use your slope from the previous question ($0.0539\ N \cdot m^2$) and $k = 8.99 \times 10^9\ N \cdot m^2/C^2$ to find q_2, given $q_1 = 2.0 \times 10^{-6}$ C. Explain what the line would show about the force if q_2 were doubled.

1. Givens: slope = $kq_1q_2 = 0.0539\ N \cdot m^2$; $q_1 = 2.0 \times 10^{-6}$ C. Question: q_2, and the effect of doubling q_2.

2. q_2 = slope ÷ $(kq_1) = 0.0539 \div ((8.99 \times 10^9)(2.0 \times 10^{-6})) = 3.00 \times 10^{-6}$ C.

3. If q_2 doubled, the slope kq_1q_2 would double, so the line would be twice as steep.

B6 · Potential difference: energy per charge

Completed notes

Explain potential difference as energy per charge, use $\Delta E = q\Delta V$, and use the electron volt.

Slide 1 of 7 · Work that stores energy between plates

A positive particle against the **negative** plate is where it wants to be, so we call its electric potential energy (written Ep) **zero** there, like a ball at the bottom of a hill.

To carry it toward the other plate something outside must push, because the plates pull and push it **back**. Our push does **work**.

The speed stays constant, so none of the work becomes **kinetic energy**. **All** the work is stored as electric potential energy: $\Delta Ep = 6.0 \times 10^{-16}$ J in the example, where Δ means **“change in”**. When released, Ep decreases and the same amount becomes kinetic energy, written Ek.

The positive plate **pushes** a positive charge away and the negative plate **pulls** it in.

Example. A positive particle with $q = 4.0 \times 10^{-17}$ C is carried at constant speed from the negative top plate (A) to B, gaining 6.0×10^{-16} J of electric potential energy. It is released from rest and gravity is too small to matter. Which way does it move, and how much kinetic energy does it have when it reaches the top plate?

1. The positive plate repels the particle and the negative plate attracts it, so the force at B points up and the particle moves up toward the negative top plate.

2. As it moves, electric potential energy turns into kinetic energy. With no other losses, all 6.0×10^{-16} J becomes kinetic energy at the top plate.

Try it. The same positive particle is held at B, with 6.0×10^{-16} J of electric potential energy, and is carried at constant speed back to the negative top plate at A. Does its electric potential energy increase or decrease, by how much, and what is its potential energy at A?

1. The field pushes the positive particle toward the negative plate, so the field does positive work as it moves from B to A.

2. The stored energy decreases by 6.0×10^{-16} J ($\Delta Ep = -6.0 \times 10^{-16}$ J).

3. At A, $Ep = 6.0 \times 10^{-16}$ J $- 6.0 \times 10^{-16}$ J $= 0$, which is the zero point we chose.

Slide 2 of 7 · Potential difference: energy per charge

A bigger charge stores more energy, so divide out the charge. Electric potential difference is $\Delta V = \mathbf{\Delta E \div q}$: ΔE is the **energy change** (J) and q is the **size of the charge** (C).

Its unit is **J/C** (joules per coulomb). ΔV belongs to the **plates**, not the particle: a charge three times as big stores three times the energy, and $\Delta E \div q$ gives the **same** ΔV.

Rearrange for the unknown: $\Delta E = \mathbf{q\Delta V}$. Put in the size of the charge, then ask which way it moves: moving toward the plate that attracts it means it **gains kinetic energy**. If the plates do not change, ΔV stays the **same** for a new charge.

On a graph of ΔE against q, $\Delta E = q\Delta V$ matches $y = mx$, so the slope is **ΔV**. The line passes through the **origin**, because zero charge stores zero energy. Check powers of ten, since the charges are very small.

Example. On other plates, a charge $q = 6.0 \times 10^{-18}$ C stores 1.8×10^{-16} J when moved from A to B. Find the potential difference between A and B, then find the energy stored when a charge of 1.5×10^{-17} C is moved between the same two points.

1. $\Delta V = \Delta E \div q = (1.8 \times 10^{-16}\ \text{J}) \div (6.0 \times 10^{-18}\ \text{C}) = 30\ \text{J/C}$.

2. The plates are the same, so $\Delta V = 30$ J/C. $\Delta E = q\Delta V = (1.5 \times 10^{-17}\ \text{C})(30\ \text{J/C}) = 4.5 \times 10^{-16}$ J.

Try it. The same plates move several charges from A to B, and the energy stored is graphed against the charge. The line passes through $C = (2.0 \times 10^{-18}\ \text{C}, 0.60 \times 10^{-16}\ \text{J})$ and $D = (8.0 \times 10^{-18}\ \text{C}, 2.40 \times 10^{-16}\ \text{J})$. What does the slope represent, and what is its value?

1. Match $\Delta E = q\Delta V$ to $y = mx$. ΔE is vertical, q is horizontal, so the slope is the potential difference ΔV.

2. Slope $= (2.40 \times 10^{-16} - 0.60 \times 10^{-16})\ \text{J} \div (8.0 \times 10^{-18} - 2.0 \times 10^{-18})\ \text{C} = 1.8 \times 10^{-16} \div (6.0 \times 10^{-18}) = 30$ J/C.

3. The line passes through the origin because zero charge stores zero energy.

Slide 3 of 7 · Method: how to solve an energy-per-charge problem

Step 1: name the **unknown**. Is it ΔE, q or ΔV? Write it before any number.

Step 2: write the equation with the unknown alone: ΔV = **ΔE/q**, ΔE = **qΔV** or q = **ΔE/ΔV**.

Step 3: put in the **sizes**: the size of the charge in coulombs and the energy in joules. Then ask which way it moves: toward the plate that attracts it, it gains **kinetic energy**.

Step 4: calculate with units. Joules per coulomb is the **volt**. Check: a bigger charge stores **more** energy.

Example. A charge of 6.0×10^{-18} C stores 1.8×10^{-16} J when it is moved from A to B. A charge of 1.5×10^{-17} C is then moved between the same two points. Find the potential difference, then the energy stored for the new charge.

1. Steps 1 and 2: part one wants ΔV, so ΔV = ΔE ÷ q. Part two wants ΔE, so ΔE = qΔV.

2. Step 3: ΔV = $(1.8 \times 10^{-16}$ J) ÷ $(6.0 \times 10^{-18}$ C). Then ΔE = $(1.5 \times 10^{-17}$ C) × ΔV, using the sizes of the charges.

3. Step 4: ΔV = 30 J/C, so ΔE = $(1.5 \times 10^{-17})(30) = 4.5 \times 10^{-16}$ J. The unit J/C is the volt, so ΔV = 30 V.

Try it. A charge of 2.5×10^{-17} C stores 1.0×10^{-15} J when it is moved between two plates. Find the potential difference between the plates, then the energy stored when a charge of 8.0×10^{-18} C is moved between the same plates.

1. Steps 1 and 2: first ΔV = ΔE ÷ q, then ΔE = qΔV.

2. Step 3: ΔV = $(1.0 \times 10^{-15}$ J) ÷ $(2.5 \times 10^{-17}$ C).

3. Step 4: ΔV = 40 J/C. Then ΔE = $(8.0 \times 10^{-18}$ C)(40 J/C) = 3.2×10^{-16} J.

4. Check: the smaller charge stores less energy than the first charge.

Slide 4 of 7 · The volt: one joule per coulomb

One volt is one **joule per coulomb**: 1 V = **1 J/C**. “Voltage” is just **potential difference**. A battery label such as 9.0 V means ΔV = **9.0 V** in an equation. Unlike the field E, ΔV has **no direction**.

A 9.0 V battery gives **9.0 J** to each coulomb that passes through it. A battery does not store charge to hand out; it supplies **energy** for each coulomb already in the wires.

Pick the form that matches the unknown: energy from charge, ΔE = **qΔV**; charge from energy, q = **ΔE ÷ ΔV**. The unit V can always be swapped for **J/C**.

Example. A battery is labelled 9.0 V and is connected to a heating coil. A charge of 4.0 C flows through the coil. Find the energy delivered.

1. ΔE = qΔV = (4.0 C)(9.0 V) = 36 J.

2. That energy shows up as heat in the coil.

Try it. A 6.0 V battery is connected to a lamp. How much energy is delivered when 2.5 C of charge passes through the lamp, and how much charge must pass to deliver 90 J?

1. ΔE = qΔV = (2.5 C)(6.0 V) = 15 J.

2. q = ΔE ÷ ΔV = 90 J ÷ 6.0 V = 15 C.

3. Check: 6.0 J per coulomb, so 90 J needs many coulombs.

Slide 5 of 7 · A charge accelerated through a voltage

Voltage supplies energy and the particle turns it into speed. Rearrange $\Delta V = \Delta E/q$ to $\Delta E = q\Delta V$. Kinetic energy is $E_k = \frac{1}{2}mv^2$; the small i means **initial** and f means **final**. Only the electric force changes the energy here, so all of $q\Delta V$ becomes kinetic energy: $q\Delta V = \mathbf{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$.

The initial term drops out only when the problem says **“from rest”** ($v_i = \mathbf{0}$). Do not drop it automatically.

From rest: $v = \mathbf{\sqrt{(2 \times E_k \div m)}}$, where $E_k = q\Delta V$. If it is already moving, find the **start** energy $\frac{1}{2}mv_i^2$ and the **gain** $q\Delta V$. Final E_k = start + gain, then solve for v.

Use the particle’s own q and m (electron, proton and alpha particle differ). Check: a particle sped up by the voltage must end **faster** than it started.

Example. An electron starts from rest at the negative plate of an electron gun and is accelerated through $\Delta V = 1.80 \times 10^{3}$ V. With $q = 1.60 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg, find its speed at the positive plate.

1. $q\Delta V = (1.60 \times 10^{-19}\ \text{C})(1.80 \times 10^{3}\ \text{V}) = 2.88 \times 10^{-16}$ J.

2. It starts from rest, so $\frac{1}{2}mv^2 = 2.88 \times 10^{-16}$ J.

3. $v = \sqrt{(2 \times 2.88 \times 10^{-16} \div (9.11 \times 10^{-31}))} = 2.51 \times 10^{7}$ m/s.

Try it. An electron enters the gap at $v_i = 1.50 \times 10^{7}$ m/s and is accelerated through $\Delta V = 6.00 \times 10^{2}$ V. Find its final speed. Use $q = 1.60 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg.

1. Initial energy: $\frac{1}{2}mv_i^2 = \frac{1}{2}(9.11 \times 10^{-31})(1.50 \times 10^{7})^2 = 1.024 \times 10^{-16}$ J.

2. Energy gained: $q\Delta V = (1.60 \times 10^{-19})(6.00 \times 10^{2}) = 9.600 \times 10^{-17}$ J.

3. Final energy $= 1.024 \times 10^{-16} + 9.600 \times 10^{-17} = 1.984 \times 10^{-16}$ J.

4. $v_f = \sqrt{(2 \times 1.984 \times 10^{-16} \div (9.11 \times 10^{-31}))} = 2.09 \times 10^{7}$ m/s.

5. Check: faster than 1.50×10^{7} m/s. (Treating it as starting from rest gives 1.45×10^{7} m/s, a typical wrong choice.)

Slide 6 of 7 · Method: how to find the speed of a charge accelerated through a voltage

Step 1: write the energy rule **$qΔV = ½mv_f^2 − ½mv_i^2$**. The energy gained from the voltage becomes **kinetic energy**.

Step 2: find the energy **gained**: $qΔV$, with the size of the charge in coulombs and the voltage in volts. The answer is in **joules**.

Step 3: find the **starting** energy. “From rest” means $½mv_i^2 = \mathbf{0}$. If it is already moving, work out $½mv_i^2$ with the particle’s own **mass**.

Step 4: final Ek = start + **gain**. Then $v = \mathbf{\sqrt{(2 \times Ek \div m)}}$. Check: the unit is **m/s**, and a particle sped up by the voltage ends **faster** than it began.

Example. An electron starts from rest and crosses a gap with a voltage of 1.80×10^3 V. With $q = 1.60 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg, find its speed at the end.

1. Step 1: $qΔV = ½mv_f^2 − ½mv_i^2$.

2. Step 2: $qΔV = (1.60 \times 10^{-19}\ \text{C})(1.80 \times 10^3\ \text{V}) = 2.88 \times 10^{-16}$ J. Step 3: it starts from rest, so the starting energy is 0.

3. Step 4: final Ek = 2.88×10^{-16} J. $v = \sqrt{(2 \times 2.88 \times 10^{-16} \div (9.11 \times 10^{-31}))} = 2.5145\ldots \times 10^7$ m/s, so $v = 2.51 \times 10^7$ m/s.

Try it. A proton starts from rest and is accelerated through 2.00×10^3 V. With $q = 1.60 \times 10^{-19}$ C and $m = 1.67 \times 10^{-27}$ kg, find its final speed.

1. Step 1: $qΔV = ½mv_f^2 − ½mv_i^2$.

2. Step 2: $qΔV = (1.60 \times 10^{-19}\ \text{C})(2.00 \times 10^3\ \text{V}) = 3.20 \times 10^{-16}$ J. Step 3: from rest, so the starting energy is 0.

3. Step 4: $v = \sqrt{(2 \times 3.20 \times 10^{-16} \div (1.67 \times 10^{-27}))} = 6.1906\ldots \times 10^5$ m/s, so $v = 6.19 \times 10^5$ m/s.

4. Check: the unit is m/s, and the proton has much more mass than an electron, so it ends slower than 2.51×10^7 m/s.

Slide 7 of 7 · The electron volt

The electron volt (eV) is a unit of **energy**, not of voltage. One eV is the energy an electron gains when accelerated through **one volt**.

1 eV = $q\Delta V$ = $(1.60 \times 10^{-19}\ \text{C})(1\ \text{V})$ = **1.60×10^{-19} J** (data sheet).

A charge of n electrons gains **n eV** for each volt: an electron through 24.0 V gains **24.0** eV, an alpha particle (charge 2e) through 15.0 V gains **30.0** eV.

To switch units: eV × 1.60×10^{-19} = **joules**, and joules ÷ (1.60×10^{-19}) = **eV**. The equation $\Delta E = q\Delta V$ still needs joules, coulombs and volts. Read the unit in the question first and finish in it.

Example. An alpha particle (charge 3.20×10^{-19} C) is accelerated from rest through 45.0 V. Find the kinetic energy it gains, first in electron volts and then in joules.

1. Its charge is 2e, so each volt gives it 2 eV: 2 × 45.0 V = 90.0 eV.

2. In joules: 90.0 eV × 1.60×10^{-19} J/eV = 1.44×10^{-17} J.

3. Check: $q\Delta V = (3.20 \times 10^{-19}\ \text{C})(45.0\ \text{V}) = 1.44 \times 10^{-17}$ J. The two routes agree.

Try it. An electron enters the gap at 1.0×10^{6} m/s and is accelerated through 80.0 V. Find its final kinetic energy in electron volts. Record the answer as $a.bc \times 10^{n}$ eV. Use $m = 9.11 \times 10^{-31}$ kg.

1. Gain: an electron through 80.0 V gains 80.0 eV.

2. Initial energy: $\frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31}\ \text{kg})(1.0 \times 10^{6}\ \text{m/s})^2 = 4.555 \times 10^{-19}$ J, and $4.555 \times 10^{-19} \div (1.60 \times 10^{-19}) = 2.847$ eV.

3. Final energy = 80.0 eV + 2.847 eV = 82.847 eV, which rounds to 82.8 eV.

4. Record 8.28×10^{1} eV. Round only at the end.

B6 · Worked practice

Potential difference: energy per charge | Questions 1-2 of 7

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}\ N \cdot m^2/C^2$, $e = 1.60 \times 10^{-19}\ C$, electron mass $m_e = 9.11 \times 10^{-31}\ kg$, proton mass $mp = 1.67 \times 10^{-27}\ kg$, $g = 9.81\ m/s^2$.

Easy · B6-N01. A charge of 5.0×10^{-18} C is moved between two points, and 1.5×10^{-16} J of energy is stored. Find the potential difference.

1. Givens: $q = 5.0 \times 10^{-18}$ C; $\Delta E = 1.5 \times 10^{-16}$ J. Question: ΔV.

2. $\Delta V = \Delta E \div q$.

3. $\Delta V = 1.5 \times 10^{-16} \div (5.0 \times 10^{-18}) = 30.0$ V.

4. Answer: 30 V

Easy · B6-N02. A 12 V battery drives 8.0 C of charge through a motor. How much energy does the battery deliver, in joules?

1. Givens: $\Delta V = 12$ V; $q = 8.0$ C. Question: ΔE.

2. $\Delta E = q\Delta V = (8.0)(12) = 96$ J.

3. Answer: 96 J

B6 · Worked practice

Potential difference: energy per charge | Questions 3-4 of 7

Easy · B6-N03. The potential difference between two points is 30 V. A second charge, twice as large, is moved between the same points. What is the new potential difference?

A. 60 V, because the charge doubled
B. 15 V, because q is in the denominator
C. Zero, because the energy cancels
D. 30 V, because ΔV belongs to the points and not to the charge

1. Givens: ΔV = 30 V for the first charge; the charge doubles. Question: the new ΔV.

2. The energy doubles, but ΔV = ΔE/q stays the same because both double.

3. Answer: D. 30 V, because ΔV belongs to the points and not to the charge

Medium · B6-N04. An electron is accelerated from rest through 3500 V. Use e = 1.60 × 10^{-19} C and electron mass m_e = 9.11 × 10^{-31} kg. Find its final speed.

1. Givens: v_0 = 0; ΔV = 3500 V; q = 1.60 × 10^{-19} C; m = 9.11 × 10^{-31} kg. Question: v.

2. $q\Delta V = \frac{1}{2}mv^2$, so $v = \sqrt{(2q\Delta V/m)}$.

3. $v = \sqrt{(2(1.60 \times 10^{-19})(3500) \div (9.11 \times 10^{-31}))} = 3.51 \times 10^7$ m/s.

4. This is well below the speed of light, so the formula $\frac{1}{2}mv^2$ is reasonable.

5. Answer: 3.51 × 10^7 m/s

B6 · Worked practice

Potential difference: energy per charge | Questions 5-6 of 7

Medium · B6-N05. A particle with charge 4.80×10^{-19} C is accelerated from rest through 25.0 V. Find the kinetic energy it gains, in electron volts.

1. Givens: $q = 4.80 \times 10^{-19}$ C $= 3e$; $\Delta V = 25.0$ V. Question: the energy gained in eV.

2. The charge is 3e, so it gains 3 eV for each volt.

3. $3 \times 25.0 = 75.0$ eV.

4. Answer: 75.0 eV

Challenging · B6-N06. An electron enters a gap moving at 2.0×10^{7} m/s and is accelerated through 4.00×10^{2} V. Find its final speed.

1. Givens: $v_0 = 2.0 \times 10^{7}$ m/s; $\Delta V = 4.00 \times 10^{2}$ V; $q = 1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg. Question: v.

2. The electron already has kinetic energy: $\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + q\Delta V$.

3. $v = \sqrt{(v_0^2 + 2q\Delta V/m)} = \sqrt{(4.000 \times 10^{14} + 1.405 \times 10^{14})} = 2.32 \times 10^{7}$ m/s. Keep the full values on your calculator.

4. The speed rises but is not more than the starting speed squared plus the gain, so it is reasonable. Do not add the speeds directly.

5. Answer: 2.32×10^{7} m/s

B6 · Worked practice

Potential difference: energy per charge | Questions 7-7 of 7

Challenging · B6-N07. An electron and a proton are each accelerated from rest through the same 100 V. Which gains more kinetic energy, and which ends with the greater speed? Explain.

1. Givens: same $\Delta V = 100$ V; equal size of charge e; masses 9.11×10^{-31} kg and 1.67×10^{-27} kg. Question: compare energy and speed.

2. Both gain the same energy, $q\Delta V = 100$ eV, because the charge magnitude and voltage are equal.

3. From $\frac{1}{2}mv^2 = q\Delta V$, $v = \sqrt{(2q\Delta V/m)}$. The electron has the smaller mass, so it ends with the greater speed.

B7 · Fields between plates

Completed notes

Use E = ΔV/Δd between parallel plates and find force, acceleration and speed of a charge.

Slide 1 of 5 · Radial and uniform fields

A field is a region where a **test charge** feels a force. The arrows show the force on a **positive** test charge.

Around a point charge the field is **radial**: arrows point **away from** a positive source and **toward** a negative one. Closer in, the arrows bunch up: a **stronger** field. The direction changes from place to place.

Between parallel plates the field is **uniform** (constant): parallel, evenly spaced arrows, the same at R, at S and between. The arrows point from the **positive** plate to the **negative** plate.

The force on a charge is F = **qE**, and F = ma links it to the acceleration. If E does not change along the path, neither does the **acceleration**.

Example. A small positive object is released from rest in each of two fields, a point-charge field (positive source) and the uniform field between plates, and nothing else acts on it. In each field, does its acceleration increase, decrease or stay the same as it moves, and what happens to its kinetic energy?

1. Point charge: the object is pushed away and the field is weaker the farther it goes, so the force and the acceleration decrease. It still speeds up the whole way.

2. Plates: the field is uniform, so the force and the acceleration stay constant.

3. In both cases the object loses electric potential energy and gains the same amount of kinetic energy.

Try it. Describe the direction and spacing of the field arrows (a) around a negative point charge and (b) between two plates when the top plate is positive. Sketch both.

1. (a) Radial arrows pointing toward the negative charge, bunched closer near it, so the field is stronger there.

2. (b) Parallel, evenly spaced arrows pointing down from the positive top plate to the negative bottom plate, so the field is uniform.

Slide 2 of 5 · The field between plates

A uniform field is the same everywhere between plates. A bigger **voltage** or a smaller **gap** makes a **stronger** field.

Formula: $E = \mathbf{\Delta V/\Delta d}$. E is the field strength, in **V/m** (size only). ΔV is the potential difference between the plates, in **volts**. Δd is the gap between the plates, in **metres**.

Direction: from the **positive** plate to the **negative** plate. V/m and **N/C** are the same unit. Multiply both sides by Δd: $\Delta V = \mathbf{E\Delta d}$.

Plates only. For one **point charge** use $E = kQ/r^2$, where Q is the source charge and r is the distance from it. Write each rearranged form out.

On a graph of E against 1/Δd, $E = \Delta V \times (1/\Delta d)$ matches y = mx: y is **E**, x is **1/Δd** and the slope is **ΔV**. Rewrite the equation to match y = mx first.

Example. Plates are 0.0150 m apart and connected to a 120 V battery. Find the field strength between the plates, then find the battery voltage needed for a field three times as strong with the same gap.

1. $E = \Delta V \div \Delta d = 120\ V \div 0.0150\ m = 8.00 \times 10^3\ V/m$.

2. Three times the field is 2.40×10^4 V/m. $\Delta V = E\Delta d = (2.40 \times 10^4\ V/m)(0.0150\ m) = 360\ V$.

3. Check: three times 120 V. Same gap, three times the field needs three times the voltage.

Try it. The battery voltage is kept the same while the plate gap is changed. The field is graphed against 1/Δd. The line passes through C = ($25\ m^{-1}$, 4.5×10^3 V/m) and D = ($100\ m^{-1}$, 18.0×10^3 V/m). What does the slope represent, and what is its value?

1. $E = \Delta V(1/\Delta d)$ matches y = mx, so the slope is the potential difference ΔV.

2. Slope = $(18.0 \times 10^3 - 4.5 \times 10^3)\ V/m \div (100 - 25)\ m^{-1} = 1.35 \times 10^4 \div 75 = 180\ V$.

Slide 3 of 5 · Force, acceleration and speed between plates

Force gives **acceleration**. Energy gives **speed**. Only the electric force acts, so ma = **qE**. From B4: F = qE, where q is the particle's charge.

Field: E = **ΔV/Δd**. Acceleration: a = **qE/m**. Speed from rest: qΔV = **½mv²**, so v = **√(2qΔV/m)**. The field is uniform, so a is **constant**.

If the question gives E and not ΔV, find ΔV = **EΔd** first. Use the electron's charge and mass from the data sheet. Keep every digit on your calculator and round **once**.

Use the particle's own q and m. A proton has the same size of charge as an electron but a much bigger **mass**. Two routes that agree are a good check. A speed stays well below **3×10^8 m/s**.

Example. An electron ($q = 1.60 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) starts from rest in a uniform field of 4.00×10^3 V/m. The plates are 0.0600 m apart. Find the final speed of the electron at the other plate.

1. $\Delta V = E\Delta d = (4.00 \times 10^3 \text{ V/m})(0.0600 \text{ m}) = 240$ V.

2. From rest: $q\Delta V = \frac{1}{2}mv^2$, so $v = \sqrt{(2q\Delta V/m)}$.

3. $v = \sqrt{(2 \times 1.60 \times 10^{-19} \times 240 \div (9.11 \times 10^{-31}))} = 9.18 \times 10^6$ m/s.

Try it. A proton ($q = 1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) starts from rest against the positive plate. The plates are 0.0150 m apart and the voltage is 300 V. Find its final speed using energy, then check the speed using the acceleration and the gap.

1. Energy: $v = \sqrt{(2q\Delta V/m)} = \sqrt{(2 \times 1.60 \times 10^{-19} \times 300 \div (1.67 \times 10^{-27}))} = 2.40 \times 10^5$ m/s.

2. Force: $E = \Delta V/\Delta d = 2.00 \times 10^4$ V/m. $a = qE/m = 1.92 \times 10^{12}$ m/s².

3. $v = \sqrt{(2a\Delta d)} = 2.40 \times 10^5$ m/s. The two routes agree.

Slide 4 of 5 · Method: parallel-plate problems

Step 1: find the field. E = **ΔV/Δd**, with the gap in **metres**.

Step 2: find the field direction. It points from the **positive** plate to the **negative** plate.

Step 3: find the force. F = **qE**, using the size of q. A positive charge is pushed **along** the field. A negative charge is pushed **against** it.

Step 4: find the acceleration. a = **F ÷ m**, in the same direction as the force. Step 5: find the speed. From rest, qΔV = **½mv²**, or check with v = **√(2aΔd)**. Check: a speed stays well below **3 × 10⁸ m/s**.

Example. An alpha particle ($q = 3.20 \times 10^{-19}$ C, $m = 6.65 \times 10^{-27}$ kg) starts from rest against the positive plate. The gap is 0.0200 m and the battery is 175 V. Find its acceleration and its speed at the other plate.

1. Step 1: $E = \Delta V/\Delta d = 175 \div 0.0200 = 8.75 \times 10^{3}$ V/m. Step 2: the field points from the positive plate to the negative plate.

2. Step 3: $F = qE = (3.20 \times 10^{-19})(8.75 \times 10^{3}) = 2.800 \times 10^{-15}$ N, along the field because the charge is positive.

3. Step 4: $a = F \div m = 2.800 \times 10^{-15} \div (6.65 \times 10^{-27}) = 4.2105\ldots \times 10^{11}$ m/s², which is 4.21×10^{11} m/s².

4. Step 5: $v = \sqrt{(2a\Delta d)} = \sqrt{(2 \times 4.2105\ldots \times 10^{11} \times 0.0200)} = 1.30 \times 10^{5}$ m/s. The energy route gives the same speed.

Try it. An electron ($q = 1.60 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) starts from rest against the negative plate. The gap is 0.0300 m and the battery is 450 V. Find its acceleration and its speed at the other plate. Write each step number beside your work.

1. Step 1: $E = 450 \div 0.0300 = 1.50 \times 10^{4}$ V/m. Step 2: the field points from the positive plate to the negative plate, toward the electron’s starting plate.

2. Step 3: $F = qE = (1.60 \times 10^{-19})(1.50 \times 10^{4}) = 2.40 \times 10^{-15}$ N. The charge is negative, so the force is against the field, toward the positive plate.

3. Step 4: $a = 2.40 \times 10^{-15} \div (9.11 \times 10^{-31}) = 2.6345\ldots \times 10^{15}$ m/s².

4. Step 5: $v = \sqrt{(2 \times 2.6345\ldots \times 10^{15} \times 0.0300)} = 1.26 \times 10^{7}$ m/s. Check: well below 3×10^{8} m/s.

Slide 5 of 5 · Not starting from rest

If a particle is already moving when it enters (look for "**enters at**"), it has initial kinetic energy ½mv_i², and the field adds more or takes some away.

Find ΔV = **EΔd**. Find the initial kinetic energy from the entering speed. Find the energy gained or lost: **qΔV**.

An electron moving toward the + plate: unlike charges **attract**, so it **speeds up**. Ek final = Ek initial **+ qΔV**. A proton moving toward the + plate: like charges **repel**, so it **slows down**. Ek final = Ek initial **− qΔV**.

Then v_f = **√(2Ek_final/m)**. Check: a charge that speeds up ends **faster**, and a slowed charge ends **slower**, than its starting speed.

Example. An electron ($q = 1.60 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) enters the gap at 2.00×10^{6} m/s toward the positive plate. The field is 3.00×10^{3} V/m and the gap is 0.0500 m. Find its speed at the positive plate.

1. $\Delta V = E\Delta d = (3.00 \times 10^{3}\ \text{V/m})(0.0500\ \text{m}) = 150$ V.

2. Initial kinetic energy = $\frac{1}{2}(9.11 \times 10^{-31})(2.00 \times 10^{6})^2 = 1.82 \times 10^{-18}$ J. Energy gained = $q\Delta V = 2.40 \times 10^{-17}$ J.

3. Final kinetic energy = 2.58×10^{-17} J.

4. $v = \sqrt{(2 \times 2.58 \times 10^{-17} \div (9.11 \times 10^{-31}))} = 7.53 \times 10^{6}$ m/s, which is faster than 2.00×10^{6} m/s.

Try it. A proton ($q = 1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) enters the gap moving at 4.0×10^{5} m/s toward the positive plate, which repels it. The potential difference is 400 V. Find its speed at the positive plate, and say how you know the answer makes sense.

1. Initial kinetic energy = $\frac{1}{2}(1.67 \times 10^{-27})(4.0 \times 10^{5})^2 = 1.34 \times 10^{-16}$ J.

2. The positive plate repels the proton, so it gives up $q\Delta V = (1.60 \times 10^{-19})(400) = 6.40 \times 10^{-17}$ J.

3. Final kinetic energy = $1.336 \times 10^{-16} - 6.40 \times 10^{-17} = 6.96 \times 10^{-17}$ J.

4. $v = \sqrt{(2 \times 6.96 \times 10^{-17} \div (1.67 \times 10^{-27}))} = 2.89 \times 10^{5}$ m/s.

5. Check: it is less than 4.0×10^{5} m/s, as it should be for a slowed charge.

B7 · Worked practice

Fields between plates | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass $m_p = 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B7-N01. Two parallel plates 0.0300 m apart are connected to a 150 V battery. Find the electric field strength between them in V/m.

1. Givens: $\Delta d = 0.0300$ m; $\Delta V = 150$ V. Question: E.

2. $E = \Delta V \div \Delta d = 150 \div 0.0300 = 5000$ V/m (5000 V/m).

3. Answer: 5000 V/m

Easy · B7-N02. The gap between two plates is halved while the battery voltage stays the same. What happens to the field strength between them?

A. It doubles.
B. It halves.
C. It stays the same.
D. It becomes four times larger.

1. Givens: ΔV constant; $\Delta d \div 2$. Question: change in E.

2. $E = \Delta V/\Delta d$, so dividing Δd by 2 multiplies E by 2.

3. Answer: A. It doubles.

B7 · Worked practice

Fields between plates | Questions 3-4 of 8

Easy · B7-N03. A positive charge is released from rest at different places between large parallel plates. How does its acceleration compare from place to place?

A. It is larger near the positive plate.
B. It is larger near the negative plate.
C. It increases as the charge speeds up.
D. It is the same everywhere, because the field is uniform.

1. Givens: a uniform field. Question: how a changes.

2. $F = qE$ with the same E everywhere, so F and $a = F/m$ are constant.

3. Answer: D. It is the same everywhere, because the field is uniform.

Medium · B7-N04. An electron starts from rest in a uniform field of 2500 V/m across plates 0.08 m apart. Find its final speed.

1. Givens: $E = 2500$ V/m; $\Delta d = 0.08$ m; $q = 1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg; $v_0 = 0$. Question: v.

2. First the potential difference: $\Delta V = E\Delta d = (2500)(0.08) = 200$ V.

3. Then $q\Delta V = \frac{1}{2}mv^2$: $v = \sqrt{(2q\Delta V/m)} = 8.38 \times 10^6$ m/s.

4. Answer: 8.38×10^6 m/s

B7 · Worked practice

Fields between plates | Questions 5-6 of 8

Medium · B7-N05. A proton starts at rest between plates 0.0400 m apart with a potential difference of 800 V. Use e = 1.60 × 10^{-19} C and proton mass mp = 1.67 × 10^{-27} kg. Find its acceleration in m/s².

1. Givens: ΔV = 800 V; Δd = 0.0400 m; q = 1.60 × 10^{-19} C; m = 1.67 × 10^{-27} kg. Question: a.

2. E = ΔV/Δd = 800 ÷ 0.0400 = 2.00 × 10^{4} V/m.

3. F = qE = 3.20 × 10^{-15} N.

4. a = F ÷ m = 1.92 × 10^{12} m/s².

5. Answer: 1.92 × 10^{12} m/s²

Medium · B7-N06. Using the proton in the previous question, find its final speed when it reaches the far plate.

1. Givens: a = 1.92 × 10^{12} m/s²; Δd = 0.0400 m; v_0 = 0. Question: v.

2. $v^2 = v_0^2 + 2ad$.

3. v = √(2(1.92 × 10^{12})(0.0400)) = 3.92 × 10^{5} m/s.

4. Check with energy: √(2qΔV/m) gives the same value.

5. Answer: 3.92 × 10^{5} m/s

B7 · Worked practice

Fields between plates | Questions 7-8 of 8

Challenging · B7-N07. An electron enters the gap at 1.0×10^{6} m/s toward the positive plate. The plates are 0.020 m apart with 3.0×10^{3} V across them. Find its speed at the positive plate.

1. Givens: $v_0 = 1.0 \times 10^{6}$ m/s; $\Delta d = 0.020$ m; $\Delta V = 3.0 \times 10^{3}$ V; $q = 1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg. Question: v.

2. $a = q\Delta V/(m\Delta d) = 2.6 \times 10^{16}$ m/s².

3. $v = \sqrt{(v_0^2 + 2ad)} = \sqrt{(1.0 \times 10^{12} + 1.05 \times 10^{15})} = 3.2 \times 10^{7}$ m/s.

4. The electron started with speed, so we keep v_0^2 in the equation.

5. Answer: 3.2×10^{7} m/s

Challenging · B7-N08. A graph of E against $1/\Delta d$ for a fixed battery is a straight line through the origin with slope 150 V. What does the slope represent, and what happens to E if the gap is doubled?

1. Givens: E vs $1/\Delta d$, slope 150 V. Question: meaning of the slope; effect of doubling the gap.

2. $E = \Delta V(1/\Delta d)$ matches $y = mx$, so the slope is the battery voltage, 150 V.

3. Doubling Δd halves $1/\Delta d$, so E is halved.

B8 · Energy conservation and 2-D motion of charges

Completed notes

Use energy conservation, then split a sideways-entering charge into constant-speed and accelerated motion.

Slide 1 of 7 · Energy conservation for a charge in a field

Ep means electric potential energy and Ek means kinetic energy, as in B6. “At A” and “at B” name the two points. Only the electric force does work, so total energy **stays the same**: Ep(A) + Ek(A) = **Ep(B) + Ek(B)**.

If the charge starts **at rest**, Ek(A) = 0, so Ek(B) = **Ep(A) − Ep(B)**. The potential energy lost becomes **kinetic energy**.

Then ½mv² = Ek, so v = **√(2Ek/m)**. Use the object’s own **mass** (the ball’s, not an electron’s).

The charge q is not needed when the energies are given. To compare speeds at two points, compare the **kinetic energies**: more kinetic energy means more speed for the same mass.

Example. A charged pith ball ($m = 2.00 \times 10^{-4}$ kg, $q = 1.5 \times 10^{-8}$ C) is held at A in a uniform field with 4.0×10^{-7} J of electric potential energy, then released. At B it has 2.5×10^{-8} J of electric potential energy. Find its speed at B.

1. It starts from rest, so Ek(A) = 0 and Ek(B) = $4.0 \times 10^{-7} - 2.5 \times 10^{-8} = 3.75 \times 10^{-7}$ J.

2. $\frac{1}{2}mv^2 = 3.75 \times 10^{-7}$ J, so $v = \sqrt{(2 \times 3.75 \times 10^{-7} \div (2.00 \times 10^{-4}))} = 6.12 \times 10^{-2}$ m/s.

3. The charge was not needed.

Try it. The same pith ball is released from rest at A, which has 4.0×10^{-7} J of electric potential energy. It reaches a point C where its electric potential energy is 1.0×10^{-7} J. Find its kinetic energy at C and say whether its speed there is greater or less than at B (where Ek = 3.75×10^{-7} J).

1. Total at A equals total at C, and it starts at rest: Ek(C) = Ep(A) − Ep(C) = 4.0×10^{-7} J − 1.0×10^{-7} J = 3.0×10^{-7} J.

2. At B the kinetic energy was 3.75×10^{-7} J, which is greater, so the ball is moving faster at B than at C.

Slide 2 of 7 · A charge entering a uniform field sideways

A charge entering a field sideways is like a ball rolling off a table: it keeps moving forward while a **force** pulls it down. Here the force is **electric**, and gravity is too small to matter. The horizontal speed v_x is the speed **along the plates**.

The electric force is at **right angles** to the plates and points toward the plate of the opposite sign (an electron toward the **positive** plate). Motion does not need a force in its own direction.

The two directions do not **affect** each other. Horizontal: net force **zero**, acceleration zero, constant velocity v_x, **uniform** motion. Vertical: constant force, constant acceleration $a = \mathbf{qE/m}$, speed grows from zero, **uniformly accelerated** motion.

Together these make a **parabola**. In explanations say “net force zero” or “constant force” in each direction.

Example. An electron enters the uniform field between parallel plates moving horizontally with speed v_x. Gravity is too small to matter. The top plate is negative and the bottom plate is positive. Complete the table for horizontal and vertical motion (net force, acceleration, velocity, motion type), then name the shape of the path.

1. Horizontal: net force zero, acceleration zero, constant velocity v_x, uniform motion.

2. Vertical: constant electric force toward the positive plate, constant acceleration, vertical speed grows from zero, uniformly accelerated motion.

3. The path is a parabola.

Try it. A proton enters the same field at the same horizontal speed as the electron, beside the top plate (negative top, positive bottom). Compare the proton’s path with the electron’s path in direction and in how sharply it bends.

1. The proton is positive, so the force on it points toward the negative plate. The electron was pushed toward the positive plate, so the two curves bend in opposite directions.

2. The charges are equal in size, so the forces are equal. The proton’s mass is much larger, so $a = qE/m$ is much smaller and its path is flatter. Both paths are parabolas.

Slide 3 of 7 · From acceleration back to voltage

To go from motion back to voltage, work the chain backward from what you want, then forward with numbers. The electric force is the only force, so F = **ma**.

Then E = **F/q**, where q is the charge that feels the force (use the electron's size, 1.60×10^{-19} C; the direction is found separately), and then $\Delta V = \mathbf{E\Delta d}$ (multiply both sides of $E = \Delta V/\Delta d$ by Δd). Each formula has three quantities, so each step is one **small calculation**, with units.

Write the chain of formulas **first**, before any numbers: a → F → E → ΔV. Use the electron's mass and charge from the data sheet.

Never use $\Delta V = \Delta Ep/q$ here: the **energy** is not given.

Example. An electron ($m = 9.11 \times 10^{-31}$ kg, $q = 1.60 \times 10^{-19}$ C) between parallel plates has an acceleration of 3.00×10^{15} m/s². The plates are 0.0360 m apart. Find the potential difference between the plates.

1. $F = ma = (9.11 \times 10^{-31})(3.00 \times 10^{15}) = 2.733 \times 10^{-15}$ N.

2. $E = F/q = 2.733 \times 10^{-15} \div (1.60 \times 10^{-19}) = 1.708 \times 10^{4}$ N/C.

3. $\Delta V = E\Delta d = (1.708 \times 10^{4})(0.0360) = 615$ V.

Try it. An electron ($m = 9.11 \times 10^{-31}$ kg, $q = 1.60 \times 10^{-19}$ C) has an acceleration of 2.50×10^{15} m/s² between plates 0.0480 m apart. Find the potential difference between the plates.

1. $F = ma = (9.11 \times 10^{-31}\text{ kg})(2.50 \times 10^{15}\text{ m/s}^2) = 2.278 \times 10^{-15}$ N.

2. $E = F/q = 2.278 \times 10^{-15} \div (1.60 \times 10^{-19}) = 1.4238 \times 10^{4}$ N/C.

3. $\Delta V = E\Delta d = (1.4238 \times 10^{4}\text{ V/m})(0.0480\text{ m}) = 683$ V.

Slide 4 of 7 · The full two-direction problem

L is the **length of the plates** and Δd is the **gap between them**. Time is the **link** between the two directions. Horizontal motion is uniform: $t = \mathbf{L/v_x}$.

Vertically $\Delta d = v_i t + \frac{1}{2}at^2$, and the vertical starting speed v_i is **0**, so $\Delta d = \mathbf{\frac{1}{2}at^2}$, which gives $a = \mathbf{2\Delta d/t^2}$. Then run the chain: $F = \mathbf{ma}$, $E = \mathbf{F/q}$, $\Delta V = \mathbf{E\Delta d}$.

“Just reaches the far end of the other plate” means the particle starts beside one plate and covers the full gap Δd (the vertical distance) in the time t. This is the greatest voltage that lets the electron escape: a bigger voltage gives a bigger acceleration, and the electron would hit the plate **first**. Set up two columns, horizontal and vertical, and fill in what you know in each.

For the final speed, the vertical speed is $v_y = \mathbf{\sqrt{(2ad)}}$ (from $v^2 = v_i^2 + 2ad$ with $v_i = 0$, and d the vertical distance); combine it with v_x as two sides of a right triangle: $v = \mathbf{\sqrt{(v_x^2 + v_y^2)}}$, angle $\theta = \tan^{-1}(\mathbf{v_y/v_x})$, measured from the **horizontal**. Name the principles in order: uniform motion, then uniform acceleration.

Example. An electron enters horizontally at $v_x = 4.00 \times 10^6$ m/s beside one plate. The plates are L = 0.100 m long and Δd = 0.0400 m apart. It just reaches the far end of the other plate. Find the potential difference between the plates. ($m = 9.11 \times 10^{-31}$ kg, $q = 1.60 \times 10^{-19}$ C)

1. $t = L/v_x = 0.100 \div (4.00 \times 10^6) = 2.50 \times 10^{-8}$ s.

2. $\Delta d = \frac{1}{2}at^2$, so $a = 2\Delta d/t^2 = 2(0.0400) \div (2.50 \times 10^{-8})^2 = 1.28 \times 10^{14}$ m/s².

3. $F = ma = 1.166 \times 10^{-16}$ N, $E = F/q = 729$ N/C.

4. $\Delta V = E\Delta d = 29.2$ V.

Try it. An electron enters horizontally at 3.00×10^6 m/s beside one plate. The plates are 0.0800 m long and 0.0300 m apart, and it just reaches the far end of the other plate. Find the potential difference between the plates, and name the two physics principles you used, in order.

1. Horizontal: $t = L/v_x = 0.0800 \div (3.00 \times 10^6) = 2.667 \times 10^{-8}$ s.

2. Vertical: $\Delta d = \frac{1}{2}at^2$, so $a = 2(0.0300) \div (2.667 \times 10^{-8})^2 = 8.4354 \times 10^{13}$, which is 8.44×10^{13} m/s².

3. $F = ma = (9.11 \times 10^{-31})(8.4354 \times 10^{13}) = 7.685 \times 10^{-17}$ N. $E = F/q = 480$ N/C. $\Delta V = E\Delta d = (480)(0.0300) = 14.4$ V.

4. Principles in order: uniform motion, then uniform acceleration.

Slide 5 of 7 · Method: how to solve a two-direction problem

Step 1, **Split**: make an x column (along the plates, **constant speed**) and a y column (across the plates, **constant acceleration**).

Step 2, **List**: write what you know in each column. A sideways entry means the across speed at the start is **0**.

Step 3, **Time**: time is the **link** between the two columns. Get it from the column with enough numbers: $t = \mathbf{L \div v_x}$.

Step 4, **Link**: across the plates, $\Delta d = \mathbf{\frac{1}{2}at^2}$, $F = \mathbf{ma}$, $E = \mathbf{F \div q}$ and $\Delta V = \mathbf{E\Delta d}$. Use the chain forwards or backwards.

Step 5, **Combine**: $v_y = \mathbf{at}$. Then $v = \mathbf{\sqrt{(v_x^2 + v_y^2)}}$ and the angle = $\mathbf{\tan^{-1}(v_y \div v_x)}$ from the horizontal.

Check: v_x never changes and v_y grows with time, so the final speed must be **bigger than v_x**.

Example. An electron enters horizontally at $v_x = 4.00 \times 10^6$ m/s beside one plate. The plates are L = 0.100 m long and $\Delta d = 0.0400$ m apart, and it just reaches the far end of the other plate. Use the five steps to find the potential difference between the plates, then the final speed and the angle. ($m = 9.11 \times 10^{-31}$ kg, $q = 1.60 \times 10^{-19}$ C)

1. Step 1: x column along the plates at constant speed. y column across the plates with constant acceleration.

2. Step 2: x has L = 0.100 m and $v_x = 4.00 \times 10^6$ m/s. y has $\Delta d = 0.0400$ m and starting across speed 0.

3. Step 3: $t = L \div v_x = 0.100 \div (4.00 \times 10^6) = 2.50 \times 10^{-8}$ s.

4. Step 4: $a = 2\Delta d \div t^2 = 1.28 \times 10^{14}$ m/s². $F = ma = 1.166\ldots \times 10^{-16}$ N. $E = F \div q = 728.8\ldots$ N/C. $\Delta V = E\Delta d = 29.2$ V.

5. Step 5: $v_y = at = 3.20 \times 10^6$ m/s. $v = \sqrt{((4.00 \times 10^6)^2 + (3.20 \times 10^6)^2)} = 5.12 \times 10^6$ m/s. Angle $= \tan^{-1}(3.20 \div 4.00) = 38.7°$ from the horizontal.

Try it. NEW, for teacher review. An electron enters horizontally at 2.50×10^6 m/s beside one plate. The plates are 0.0600 m long and 0.0150 m apart, and it just reaches the far end of the other plate. Use the five steps to find the potential difference between the plates and the final speed.

1. Step 3: $t = L \div v_x = 0.0600 \div (2.50 \times 10^6) = 2.40 \times 10^{-8}$ s.

2. Step 4: $a = 2\Delta d \div t^2 = 2(0.0150) \div (2.40 \times 10^{-8})^2 = 5.2083 \times 10^{13}$ m/s^2.

3. $F = ma = 4.7448 \times 10^{-17}$ N. $E = F \div q = 296.55$ N/C. $\Delta V = E\Delta d = 296.55 \times 0.0150 = 4.45$ V.

4. Step 5: v_y = at = 1.25×10^6 m/s.

5. $v = \sqrt{((2.50 \times 10^6)^2 + (1.25 \times 10^6)^2)} = 2.80 \times 10^6$ m/s, at $\tan^{-1}(1.25 \div 2.50) = 26.6°$ from the horizontal.

Slide 6 of 7 · Where does the particle land?

A long numerical response is worth doing in order, with each result **labelled**: field, force, acceleration, time, distance.

Field from the voltage and the gap: $E = \mathbf{\Delta V/\Delta d}$. Acceleration with the particle's own charge and mass: $a = \mathbf{qE/m}$.

Vertical motion starts at zero vertical speed and covers the gap: $\Delta d = \mathbf{\frac{1}{2}at^2}$, so $t = \mathbf{\sqrt{(2\Delta d/a)}}$. Horizontal motion is constant speed, so the horizontal distance is $x = \mathbf{v_x t}$.

Check the units at the end, then record the answer as $\mathbf{a.bc \times 10^n}$ with its unit and three digits.

Example. An alpha particle ($q = 3.20 \times 10^{-19}$ C, $m = 6.65 \times 10^{-27}$ kg) enters horizontally at 2.00×10^5 m/s beside the top plate of a pair of plates 0.0260 m apart with 600 V across them. The top plate is positive and the bottom plate is negative, so the alpha particle is pushed toward the bottom plate and lands on it. Find the horizontal distance it travels. Record the answer as $a.bc \times 10^n$ m.

1. $E = \Delta V/\Delta d = 600 \div 0.0260 = 2.31 \times 10^4$ V/m.

2. $a = qE/m = (3.20 \times 10^{-19})(2.31 \times 10^4) \div (6.65 \times 10^{-27}) = 1.11 \times 10^{12}$ m/s².

3. Vertically, $\Delta d = \frac{1}{2}at^2$, so $t = \sqrt{(2 \times 0.0260 \div (1.11 \times 10^{12}))} = 2.164 \times 10^{-7}$ s.

4. $x = v_x t = (2.00 \times 10^5)(2.164 \times 10^{-7}) = 4.328 \times 10^{-2}$ m, which is 4.33×10^{-2} m. Record 4.33×10^{-2}.

Try it. An electron ($q = 1.60 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) enters horizontally at 5.00×10^6 m/s beside the top plate of a pair of plates 0.0200 m apart with 100 V across them. The top plate is negative and the bottom plate is positive, so the electron is pulled toward the bottom plate and lands on it. Find the horizontal distance it travels.

1. $E = \Delta V/\Delta d = 100 \div 0.0200 = 5.00 \times 10^3$ V/m.

2. $a = qE/m = (1.60 \times 10^{-19})(5.00 \times 10^3) \div (9.11 \times 10^{-31}) = 8.78 \times 10^{14}$ m/s².

3. Vertically, $\Delta d = \frac{1}{2}at^2$, so $t = \sqrt{(2 \times 0.0200 \div (8.78 \times 10^{14}))} = 6.7497 \times 10^{-9}$ s.

4. $x = v_x t = (5.00 \times 10^6)(6.7497 \times 10^{-9}) = 3.3749 \times 10^{-2}$ m, which is 3.37×10^{-2} m.

Slide 7 of 7 · Still charges: the oil-drop experiment

A drop that hangs still has **no net force**. The electric force **up** equals the weight **down**: qE = **mg**. So q = **mg ÷ E**. Find the field first: E = **ΔV ÷ Δd**.

Count the extra electrons with n = **q ÷ e**, where e = **1.60 × 10⁻¹⁹ C**. Electrons are counted, so n is a **whole number**. Use g = **9.81 m/s²**.

The field points from the positive plate to the negative plate. A drop pushed **against** the field is negative. A negative drop has **extra** electrons.

Millikan found that every measured charge was a whole-number multiple of **e**: q = **n × e**. A value that gives a **fraction** when divided by e cannot be right. Charge comes in **units**.

Example. An oil drop of mass 1.96×10^{-15} kg hangs still between plates 0.0100 m apart with 400 V across them. The top plate is positive. Find the charge on the drop and the number of extra electrons. (g = 9.81 m/s², e = 1.60×10^{-19} C)

1. E = ΔV ÷ Δd = 400 ÷ 0.0100 = 4.00×10^{4} N/C.

2. mg = $(1.96 \times 10^{-15})(9.81) = 1.9228... \times 10^{-14}$ N.

3. qE = mg, so q = mg ÷ E = $1.9228... \times 10^{-14} \div (4.00 \times 10^{4}) = 4.807 \times 10^{-19}$ C.

4. n = q ÷ e = $4.807 \times 10^{-19} \div (1.60 \times 10^{-19}) = 3.004...$, so 3 extra electrons. The force is up and the field is down, so the drop is negative.

Try it. NEW, for teacher review. A drop of mass 3.26×10^{-15} kg hangs still between plates 0.0200 m apart with 500 V across them. The top plate is positive. Find the charge on the drop and the number of extra electrons. (g = 9.81 m/s², e = 1.60×10^{-19} C)

1. E = ΔV ÷ Δd = 500 ÷ 0.0200 = 2.50×10^{4} N/C.

2. mg = $(3.26 \times 10^{-15})(9.81) = 3.1981... \times 10^{-14}$ N.

3. q = mg ÷ E = $3.1981... \times 10^{-14} \div (2.50 \times 10^{4}) = 1.279... \times 10^{-18}$ C.

4. n = q ÷ e = $1.2792... \times 10^{-18} \div (1.60 \times 10^{-19}) = 7.995...$, so 8 extra electrons.

B8 · Worked practice

Energy conservation and 2-D motion of charges | Questions 1-2 of 7

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass mp $= 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B8-N01. A charged ball ($m = 3.00 \times 10^{-4}$ kg) is released from rest where its electric potential energy is 5.0×10^{-6} J. At another point its electric potential energy is 8.0×10^{-7} J. Find its speed there.

1. Givens: $v_0 = 0$; $E_{p1} = 5.0 \times 10^{-6}$ J; $E_{p2} = 8.0 \times 10^{-7}$ J; $m = 3.00 \times 10^{-4}$ kg. Question: v.

2. $E_k = E_{p1} - E_{p2} = 4.2 \times 10^{-6}$ J.

3. $v = \sqrt{(2E_k/m)} = \sqrt{(2(4.2 \times 10^{-6}) \div (3.00 \times 10^{-4}))} = 0.167$ m/s.

4. Answer: 0.167 m/s

Easy · B8-N02. A proton enters a uniform field between horizontal plates moving parallel to the plates. Which statement describes its motion?

A. Constant velocity parallel to the plates and constant acceleration across them

B. Constant acceleration in both directions

C. Constant velocity in both directions

D. Velocity changes parallel to the plates only

1. Givens: entry parallel to the plates, field across the plates. Question: the motion.

2. The force is across the plates only, so the parallel component has no force and keeps constant speed; the perpendicular component accelerates.

3. Answer: A. Constant velocity parallel to the plates and constant acceleration across them

B8 · Worked practice

Energy conservation and 2-D motion of charges | Questions 3-4 of 7

Easy · B8-N03. Name the shape of the path of a charged particle that enters a uniform electric field at right angles to the field. Explain with one sentence about each direction.

1. Givens: entry at right angles to a uniform field. Question: path shape and reason.

2. The path is a parabola.

3. Parallel to the plates the speed is constant, and across the plates the acceleration is constant, as for a ball thrown horizontally.

Medium · B8-N04. An electron has an acceleration of 4.00×10^{15} m/s² between plates 0.0250 m apart. Find the potential difference between the plates.

1. Givens: $a = 4.00 \times 10^{15}$ m/s²; $\Delta d = 0.0250$ m; $q = 1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg. Question: ΔV.

2. $F = ma = qE$, so $E = ma/q$.

3. $E = 2.2775 \times 10^{4}$ V/m, then $\Delta V = E\Delta d = 569$ V.

4. Answer: 569 V

B8 · Worked practice

Energy conservation and 2-D motion of charges | Questions 5-6 of 7

Medium · B8-N05. An electron enters horizontally at 5.00×10^{6} m/s beside one plate. The plates are 0.120 m long. How long does the electron take to cross the plates?

1. Givens: $v_{0x} = 5.00 \times 10^{6}$ m/s; $\Delta x = 0.120$ m. Question: t.

2. No horizontal force, so $\Delta x = vt$.

3. $t = \Delta x \div v = 0.120 \div (5.00 \times 10^{6}) = 2.40 \times 10^{-8}$ s.

4. Answer: 2.40×10^{-8} s

Challenging · B8-N06. Use the electron from the previous question. It just reaches the far end of the other plate, so it moves 0.0500 m across the gap during that time. Find the potential difference between the plates.

1. Givens: $t = 2.40 \times 10^{-8}$ s; $\Delta y = 0.0500$ m across the plates; $q = 1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg; $v_{y0} = 0$. Question: ΔV.

2. Vertical motion: $\Delta y = \frac{1}{2}at^2$, so $a = 2\Delta y/t^2 = 1.74 \times 10^{14}$ m/s².

3. $F = ma$, $E = F/q$, $\Delta V = E\Delta d$ with $\Delta d = 0.0500$ m: $\Delta V = 49.4$ V.

4. Answer: 49.4 V

B8 · Worked practice

Challenging · B8-N07. In the previous two questions, name the principles you used in order. Give one principle that also describes the event but you did not need.

1. Givens: the two-step problem above. Question: order of the principles and one unused principle.

2. 1) Constant horizontal velocity gave the time. 2) Constant acceleration across the plates gave a. 3) Newton’s second law with F = qE gave E, then E = ΔV/Δd gave the voltage.

3. Energy conservation also describes the event, but it would need the final speed, which this question did not give.

B9 · Magnets and magnetic fields

Completed notes

Describe poles, magnetic materials, the field around a magnet, the Earth as a magnet, and domains.

Slide 1 of 5 · Poles and magnetic materials

Every magnet has two poles: **north** and **south**. Opposite poles **attract**. Like poles **repel**.

Cut a magnet in half: each half still has **two poles**. A single pole, a **monopole**, has never been found.

An iron nail is magnetic, but it is not a magnet. It has **no poles**. It is attracted to **either** end of a magnet and is never repelled.

Strongly magnetic: **iron**, **nickel** and **cobalt**. Steel is mostly iron, so it is **magnetic** too.

On the diploma: **repelled** proves a magnet. **Attracted** does not.

Example. Bars X and Y are each brought close to both ends of a bar magnet, using the same end of each bar at both ends of the magnet. X is attracted to the north end and also attracted to the south end. Y is attracted to the north end and repelled by the south end. Which bar is a magnet, which is only magnetic, and which pole of Y is near the south end of the bar magnet?

1. X is attracted to both ends. That happens to a magnetic material without poles, such as iron, so X is magnetic but not a magnet.

2. Y is repelled by the south end. Only like poles repel, so Y is a magnet.

3. The end of Y near the south end of the bar magnet is a south pole (like poles repel).

Try it. A bar magnet is cut in half. (a) Describe the poles of each half. (b) An iron nail is attracted to the north end of a bar magnet. Is it attracted, repelled or not affected at the south end, and why?

1. (a) Each half has both a north pole and a south pole. Cutting never makes a lone pole.

2. (b) The nail is attracted at the south end too. The nail has no poles, so it is attracted to either end and is never repelled.

Slide 2 of 5 · Magnetic fields and the test compass

Every field needs a test object. Magnetism uses a small **compass**, a tiny magnet on a pivot.

The magnetic field is a **vector**: it has a size and a direction. Its symbol is **B**. Its unit is the **tesla (T)**.

Direction: the way the **north** pole of a compass points there. The field points the way the force acts on a north pole.

Outside a magnet, lines leave **north** and enter **south**. Lines are closest near the poles, where the field is **strongest**.

At any point the field touches the line (it is **tangent**) and follows the **arrow**.

Between two close opposite poles the lines are straight and parallel. The field is **uniform**: the same size and direction everywhere.

Example. A bar magnet has its north pole on the right and its south pole on the left. Compasses A and B are above and below the middle of the magnet, C is just past the north end and D is just past the south end, both on the line through the magnet. For each of A, B, C and D, give the direction its north pole points: left, right, up or down.

1. A and B (above and below the middle): left. The compass’s north pole is pulled toward the south pole on the left and pushed away from the north pole on the right.

2. C (just past the north end): right, away from the north pole.

3. D (just past the south end): right, toward the south pole.

Try it. Two bar magnets face each other with a small gap. The right end of the left magnet is a north pole and the left end of the right magnet is a south pole. In the gap, which way does the field point, what kind of field is it, and which way does a compass’s north pole point?

1. Field lines leave the north pole and enter the south pole, so the field in the gap points from the left magnet to the right magnet: right.

2. The lines are straight, parallel and evenly spaced, so the field is uniform.

3. A compass lines up along the field, so its north pole points right.

Slide 3 of 5 · Method: how to draw the field lines of a bar magnet

Step 1: mark the **poles**. Label the north end and the south end.

Step 2: draw the lines out of **N** and into **S**. Lines never **cross**.

Step 3: space the lines by **strength**. Crowd them near the poles. Parallel, evenly spaced lines mean a **uniform** field.

Step 4: add an **arrow** to every line, from N to S outside the magnet. A compass's north pole lines up that way.

Check: no line crosses another or starts in empty space.

Example. A bar magnet has its north pole on the right and its south pole on the left. Compasses A and B are above and below the middle of the magnet, C is just past the north end and D is just past the south end, both on the line through the magnet. Use the four steps to draw the field lines and say which way the north pole of each compass points.

1. Step 1: the right end is north and the left end is south.

2. Step 2: outside the magnet, every line leaves the right end, curves over or under the magnet and enters the left end.

3. Step 3: draw the lines close together near the two ends and far apart away from the magnet.

4. Step 4: put an arrow on each line from the right end around to the left end. A and B: left. C and D: right.

Try it. NEW, for teacher review. A bar magnet has its north pole on the left and its south pole on the right. Use the four steps to draw four field lines with arrows. Which way does the north pole of a compass point (a) just above the middle of the magnet and (b) just past the north end, on the line through the magnet?

1. Step 1: the left end is north and the right end is south.

2. Step 2: the lines leave the left end, curve over or under the magnet and enter the right end.

3. Step 3: crowd the lines near the two ends. Step 4: arrows run from the left end around to the right end.

4. (a) Just above the middle, the line runs left to right, so the compass points right.

5. (b) Just past the north end, the lines leave the magnet going left, away from north, so the compass points left.

Slide 4 of 5 · The Earth is a magnet

A north pole is attracted only to a **south** pole. An iron nail is attracted to **either** pole.

A compass needle’s north end points **north**, toward the top of a map. So the pole it is attracted to is up there, and it is a magnetic **south** pole.

The Earth acts like a huge bar magnet. Its south pole is near geographic **north**. Its north pole is near geographic **south**.

A compass points along the **field**, and its north end points to a **south** pole. It follows the **poles** of the magnet inside, not the labels on a model. On the diploma, the Earth’s “north” magnetic pole is really a **south** pole.

Example. A globe with a bar magnet inside stands for the Earth, with geographic north at the top. A compass C at the equator points toward the top of the page. Is the end of the Earth’s magnet inside the top of the globe a north pole or a south pole, and in which direction does the Earth’s magnetic field point at C?

1. The compass’s north pole points toward the top of the globe, and a compass’s north pole is attracted to a south pole, so the Earth’s magnet has a south pole in the top end.

2. The magnetic field at C points the same way the compass’s north end points: toward the top of the page, geographic north.

Try it. A student hides a bar magnet in a plastic ball and uses it as a model Earth. A compass T beside the ball at its equator points up toward the model’s labelled north. The compass stays where it is in the room and only the ball turns. Which pole of the hidden magnet is at the top, and what would T do if the ball were turned upside down?

1. The needle’s north end points to the top, so the top end of the hidden magnet is a south pole.

2. If the ball is flipped, the south pole is now at the bottom and the needle’s north end points down.

3. The compass follows the magnet’s poles, not the labels on the ball.

Slide 5 of 5 · Domains: why magnets and nails behave this way

A magnetic material has tiny regions called **domains**. Each is a tiny magnet with a **north** and a **south** end.

Magnet: the domains line up the **same way** and stay lined up. Plain iron nail: the domains point in **random** directions and their effects **cancel**.

Bring a south pole near the nail: free domains (they can turn) swing their **north** ends toward it. The near end becomes a **north** pole, so the nail is **attracted**. It is now a **temporary** magnet, only while a magnet is close. Plastic has **no domains** a magnet can line up.

Atoms always jiggle a little: **jostling**. When the magnet leaves, jostling turns the domains back to **random** directions. Heat and **hard knocks** make the jostling stronger and knock the domains out of line. On the diploma: use the word **domains** and link cause and effect with “because”.

Example. A magnet is pulled away from an iron nail that it had attracted. After a short time the nail no longer attracts a paper clip. Use domains to explain why the nail attracted the paper clip while the magnet was close, and why it stopped when the magnet was removed.

1. While the magnet was close, its field turned the free domains in the nail so they lined up. The nail then had a north end and a south end and could attract the paper clip.

2. After the magnet was removed, the jostling of the atoms turned the domains back to random directions. The effects cancelled, so the nail had no poles and no longer attracted the clip.

Try it. A magnet is brought near a plastic comb and near an iron nail. Explain why the nail is attracted but the comb is not, and describe two ways to weaken a permanent magnet.

1. The nail’s domains are free to turn toward the magnet, so it becomes a temporary magnet and is attracted.

2. Plastic has no domains that a magnet can line up, so the comb gets no poles and is not attracted.

3. Two ways to weaken a permanent magnet: heat it, or drop or hit it. Both jostle the domains out of line.

B9 · Worked practice

Magnets and magnetic fields | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass $m_p = 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B9-N01. A steel nail is attracted to both ends of a bar magnet. A second object is attracted to one end of the bar magnet and repelled by the other. Which statement is correct?

A. The nail is a magnet.

B. The second object is a magnet.

C. Both objects are magnets.

D. Neither object is a magnet.

1. Givens: nail attracted to both ends; object 2 attracted at one end and repelled at the other. Question: which is a magnet.

2. Repulsion only happens between two magnetic poles of the same kind, so object 2 is a magnet. A nail is attracted to either pole because it becomes a temporary magnet.

3. Answer: B. The second object is a magnet.

Easy · B9-N02. List the three pure elements that are strongly magnetic. Give one way to destroy a permanent magnet.

1. Givens: none. Question: three elements; one way to destroy a magnet.

2. Iron, cobalt and nickel.

3. Heating it strongly, hitting it repeatedly or placing it in a changing opposing field scrambles the domains.

B9 · Worked practice

Magnets and magnetic fields | Questions 3-4 of 8

Easy · B9-N03. A bar magnet has its north pole on the left. A compass is placed above the middle of the magnet. Which way does the compass’s north pole point?

A. Toward the left
B. Straight up
C. Straight down
D. Toward the right

1. Givens: N on the left, S on the right; compass above the centre. Question: direction of the compass north.

2. Outside the magnet the field runs from north to south, so above the middle it points right. The compass north follows the field.

3. Answer: D. Toward the right

Medium · B9-N04. In Edmonton, the north pole of a compass points toward the Arctic. What kind of magnetic pole does the Earth have near the Arctic? Explain.

1. Givens: a compass north pole points toward the Arctic. Question: the kind of pole near the Arctic.

2. Unlike poles attract, so the compass’s north pole is attracted to a south pole.

3. The Earth has a magnetic south pole near the geographic Arctic.

B9 · Worked practice

Magnets and magnetic fields | Questions 5-6 of 8

Medium · B9-N05. Describe the domains in (a) a bar magnet, (b) an iron nail away from any magnet, and (c) a plastic comb.

1. Givens: three objects. Question: the domains in each.

2. (a) A bar magnet has most domains lined up in the same direction.

3. (b) A nail has domains that point in random directions, so their fields cancel.

4. (c) Plastic is not a magnetic material and has no domains that line up.

Medium · B9-N06. A nail is stroked many times in one direction with the north pole of a magnet. What happens inside the nail?

A. The domains line up, so the nail becomes a weak magnet.

B. New domains are created in the nail.

C. The nail loses all of its domains.

D. The atoms in the nail change into iron.

1. Givens: a nail stroked with a magnet. Question: the change inside.

2. Stroking pulls many randomly pointing domains into the same direction. The nail has the same atoms, but it now has a net field.

3. Answer: A. The domains line up, so the nail becomes a weak magnet.

B9 · Worked practice

Magnets and magnetic fields | Questions 7-8 of 8

Challenging · B9-N07. The north pole of a bar magnet is brought near the end of a long iron nail. A second nail hangs from the first nail. What explains this?

A. The first nail becomes charged by the magnet.

B. Gravity pulls the nails together.

C. The second nail has more domains.

D. The first nail’s domains are lined up by the magnet, so it attracts the second nail.

1. Givens: a magnet near nail 1; nail 2 attracted to nail 1. Question: why nail 2 hangs.

2. The magnet’s field lines up the domains in nail 1, so nail 1 becomes a temporary magnet, lines up the domains in nail 2, and attracts it.

3. Answer: D. The first nail’s domains are lined up by the magnet, so it attracts the second nail.

Challenging · B9-N08. A bar magnet is cut in half. A student says one half is a north pole and the other half is a south pole. Correct the statement using domains.

1. Givens: a bar magnet cut in half. Question: correct the claim.

2. Each piece has lined-up domains, so each piece becomes a smaller magnet with its own north and south poles.

3. Poles cannot be isolated, so cutting never separates north from south.

B10 · Fields around currents

Completed notes

Use the right-hand rules for a straight wire, a loop and a solenoid.

Slide 1 of 6 · Currents make magnetic fields

In 1820 Oersted found that a compass near a wire turns only while the wire carries a **current**. The cause is **moving charge**: every moving charge has its own magnetic field, and in a wire the fields of billions of electrons add up. Current has symbol **I** and is measured in **amperes (A)**. One ampere is one **coulomb** of charge passing a point each **second**.

Conventional current is the way positive charge would move: out of the **positive** terminal (the positive plate of the battery), through the wire and back to the negative terminal. **Electron** flow is the real motion in a metal and points the **opposite** way.

This course uses **conventional** current for every hand rule, unless the question says otherwise.

Into the page is drawn as a circle with a **cross**. Out of the page is drawn as a circle with a **dot**. On the diploma: if a question says “electrons”, convert to conventional current **before** you apply a hand rule.

Example. A battery is in the middle of the bottom wire of a rectangular circuit, with its positive plate on the left and its negative plate on the right. Along the top wire, which way does conventional current flow, and which way do the electrons flow?

1. Conventional current leaves the positive plate on the left, goes up the left side and then along the top wire from left to right.

2. Electrons move the opposite way, from right to left along the top wire.

Try it. In a straight wire on the page, electrons flow down the page. Which way does conventional current flow? A compass beside the wire turns when the battery is connected and settles back when it is disconnected. What does that show?

1. Conventional current is opposite to the electron flow, so it flows up the page.

2. The compass turns only while there is a current. This shows that a moving charge (a current) makes a magnetic field.

Slide 2 of 6 · The right-hand rule for a straight wire

The field of a straight wire is a set of **circles** around it (look at the dot or cross picture for end-on views). Grip the wire loosely with your **right** hand, with your **thumb** along the conventional current. Your curled **fingers** show the direction of the field.

If the thumb points up the page, the fingers move across to the **right** in front of the wire and to the **left** behind it. So a point to the right of the wire has the field **into** the page and a point to the left has it **out of** the page. Thumb down reverses both. Thumb right gives **out of** the page above the wire and **into** the page below; thumb left reverses those.

The circles are closer together near the wire, so the field is **stronger** there. End-on means you look straight along the wire, so it looks like a dot or cross. With the current out of the page, the thumb points at you and the circles go **counterclockwise**.

At any point the field is **tangent** to the circle through that point. On the diploma: a wrong choice uses the **left** hand or the **electron** direction.

Example. A long wire lies in the plane of the page. Conventional current flows through it from left to right. P is a point above the wire and Q is a point below it, both in the plane of the page. At P and at Q, is the magnetic field into the page or out of the page?

1. Point the right thumb to the right, along the current.

2. At P (above the wire) the fingers curl toward you, so the field is out of the page.

3. At Q (below the wire) the fingers curl away from you, so the field is into the page.

Try it. A wire is seen end-on and its electrons flow into the page. R is a point to the right of the wire and S is a point directly above it, both in the plane of the page. Which way does the magnetic field point at R and at S: up, down, left, right, into the page or out of the page?

1. Electrons flow into the page, so conventional current is out of the page. The right thumb points toward you and the field circles are counterclockwise.

2. Picture a clock face with the wire in the middle: R is at 3 o’clock, where a counterclockwise circle goes up, so the field points up the page.

3. S is at 12 o’clock, where a counterclockwise circle goes left, so the field points left.

Slide 3 of 6 · Loops add their fields

Bend a straight wire into a loop and every piece of wire adds a field through the middle that points the **same** way. Fields are vectors, so adding them gives a **strong** field inside the loop. Outside the loop the field is weaker and points the other way. If the current in the loop is counterclockwise, the field in the middle is **out of** the page (check: on top of the loop the current goes left, so thumb left puts the field below that wire, inside the loop, out of the page).

The field B of a solenoid depends on the current I (amperes, A) and on the number of turns N. Both are direct proportions: B ∝ **I** and B ∝ **N**. The sign ∝ means “is directly proportional to”: if one doubles, the other doubles. B is in **tesla (T)**.

A **smaller** radius makes the field at the middle stronger, because the wire is **closer** to the middle. A coil of many turns is called a **solenoid**.

When two quantities change at once and both are direct proportions, the factors **multiply**. On the diploma: “in terms of B” answers look like **6B** or B/2, so write each factor out.

Example. A solenoid with a given current and number of turns makes a magnetic field of strength B at its centre. The current is doubled and the number of turns is multiplied by three. What is the new field, in terms of B?

1. The field is directly proportional to current, so doubling the current gives 2B.

2. The field is also directly proportional to the number of turns, so tripling the turns multiplies by 3 again.

3. The new field is 2 × 3 × B = 6B.

Try it. Two flat circular coils carry the same current and have the same number of turns. Coil X has radius 4.0 cm and coil Y has radius 2.0 cm. Which coil has the stronger field at its centre, and why?

1. Coil Y has the stronger field at its centre.

2. Its wire is closer to the centre, and the field of a current is stronger closer to the wire. Current and turns are the same, so the radius is the only difference.

Slide 4 of 6 · Poles of a solenoid and an electromagnet

To find the north pole of a solenoid, grip a wire on the **front** of the coil with your **right** hand, with your thumb along the **conventional** current. Look at where your fingers go on the **inside** of the coil.

The field inside points toward the **north** end, because field lines leave a north pole. If the thumb points up on the front wires, that is the “behind the wire” case of the straight-wire rule: the fingers move **left**, and inside the coil is behind the front wire. So the field inside runs left and the **left** end is the north pole.

An air-core solenoid has just air inside. An **electromagnet** has an **iron** core: the coil’s field lines up the **domains** in the iron, so the magnet is much stronger. Turn off the current and the domains go back to random.

More current, more turns or a smaller radius also make it stronger. On the diploma: the **winding** and the current direction set the pole, not the terminal label. Trace the current from the **positive** plate first.

Example. A solenoid is wound the opposite way to the one in the slide. The positive plate of the battery is on the left and the leads go to the top of the coil. In which direction does the conventional current flow in the wires on the front of the coil, and which end of the solenoid is the north pole?

1. Starting at the left (positive) lead, the current enters at the top-left and goes down through the front wires.

2. With the thumb pointing down, the fingers inside the coil point right, so the right end is the north pole.

Try it. A solenoid with an iron core is wound with its front wires slanting up. This time the positive plate of the battery is on the right. (In the first example the positive plate was on the left and the front current went up.) Which end of the electromagnet is the north pole, and what two changes would make it stronger?

1. The positive plate is on the right, so the current reverses. The front wires carry conventional current down and the field inside points right.

2. The right end is the north pole.

3. Make it stronger with more current or more turns (or a smaller radius).

Slide 5 of 6 · Method: which right-hand rule do I use?

Step 1: convert to **conventional** current. If the question gives electron flow, **reverse** it. A moving negative charge counts as current the other way.

Step 2: name the shape: a straight **wire**, one moving **charge**, or a **coil**.

Step 3: make the grip with your **right** hand. Wire or charge: **thumb** along the current. Coil: grip one **front** wire, thumb along its current.

Step 4: read the **fingers**. Wire or charge: they curl the way the field goes around it. Coil: inside the coil they show the field, and the end the field leaves is **north**.

Check: right hand, conventional current in the thumb. Electron flow in the thumb reverses your answer. On the diploma: a wrong choice uses the **left** hand or the **electron** direction.

Example. A wire is seen end-on and its electrons flow into the page. R is a point to the right of the wire and S is a point directly above it, both in the plane of the page. Which way does the magnetic field point at R and at S?

1. Step 1: electrons flow into the page, so conventional current comes out of the page.

2. Steps 2 and 3: this is a straight wire. Make the right-hand grip with the thumb pointing out of the page, toward you.

3. Step 4: the fingers curl counterclockwise. R is at 3 o’clock, where a counterclockwise circle goes up, so the field at R points up the page. S is at 12 o’clock, where it goes left, so the field at S points left.

Try it. NEW, for teacher review. In a long vertical wire on the page, electrons flow down the page. Point U is to the left of the wire, in the plane of the page. Which way does the magnetic field point at U: into the page or out of the page?

1. Step 1: electrons flow down, so conventional current flows up the page.

2. Steps 2 and 3: a straight wire. Make the right-hand grip with the thumb pointing up.

3. Step 4: with the thumb up, the fingers move across to the right in front of the wire and to the left behind it. A point to the left of the wire has the field out of the page.

4. Check: right hand, and conventional current (not electron flow) in the thumb.

Slide 6 of 6 · How much charge flows? Current is charge per time

Current is charge per time: I = **q ÷ t**, so q = **It**. I is in **amperes**, q is in **coulombs** and t must be in **seconds**.

To count electrons, divide the charge by the charge of one electron: n = **q ÷ e**, with e = **1.60×10^{-19} C**.

To go the other way, from a count to a charge, **multiply**: q = **n × e**. Then divide by the time to get the **current**.

On the diploma: the number of electrons is a **count**, so it is a whole number or a power of ten, never a fraction of an electron.

Example. A wire carries a current of 2.5 A for 4.0 s. Find the charge that passes a point in the wire and the number of electrons. ($e = 1.60 \times 10^{-19}$ C)

1. q = It = (2.5 A)(4.0 s) = 10 C.

2. n = q ÷ e = 10 ÷ (1.60×10^{-19}) = 6.25×10^{19} electrons.

Try it. NEW, for teacher review. A wire carries 3.20×10^{19} electrons past a point in 2.0 s. Find the current. ($e = 1.60 \times 10^{-19}$ C)

1. q = ne = (3.20×10^{19})(1.60×10^{-19} C) = 5.12 C.

2. I = q ÷ t = 5.12 C ÷ 2.0 s = 2.56 A.

B10 · Worked practice

Fields around currents | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass $m_p = 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B10-N01. Electrons flow through a wire from left to right. Which statement is correct?

A. Conventional current flows left to right, the same as the electrons.

B. Conventional current flows right to left, and we use the right-hand rule for conventional current.

C. Conventional current flows right to left, and we use the left-hand rule for electrons.

D. There is no current because electrons are very small.

1. Givens: electron flow left → right. Question: direction of conventional current.

2. Conventional current is the flow of positive charge, so it is opposite to the electron flow: right → left. The course uses conventional current with the right hand.

3. Answer: B. Conventional current flows right to left, and we use the right-hand rule for conventional current.

Easy · B10-N02. Conventional current flows up the page in a long vertical wire. Is the magnetic field into the page or out of the page (a) to the right of the wire, and (b) to the left of the wire?

1. Givens: current up the page. Question: field direction at the right and at the left.

2. Grip the wire with the right hand, thumb up the page. In front of the wire the fingers move to the left; behind it they move to the right.

3. So the field is into the page at a point to the right of the wire, and out of the page at a point to the left.

B10 · Worked practice

Fields around currents | Questions 3-4 of 8

Easy · B10-N03. Conventional current goes to the right in a horizontal wire in the plane of the page. At a point above the wire, the field points

A. into the page
B. up the page
C. to the right
D. out of the page

1. Givens: current right; point above the wire. Question: field direction.

2. Right thumb to the right; above the wire the fingers curl out of the page. Check: below the wire the field is into the page.

3. Answer: D. out of the page

Medium · B10-N04. Electrons flow from right to left along a long horizontal wire in the plane of the page. What is the direction of the field at a point below the wire?

1. Givens: electrons right → left. Question: field below the wire.

2. Conventional current flows left → right. Thumb to the right.

3. Below the wire the fingers curl into the page. The field is into the page.

B10 · Worked practice

Fields around currents | Questions 5-6 of 8

Medium · B10-N05. A solenoid’s current is doubled while the number of turns is cut in half (the solenoid keeps the same length). What happens to the field at the centre?

A. It doubles.

B. It stays about the same.

C. It is cut to one quarter.

D. It becomes four times larger.

1. Givens: I × 2; N ÷ 2; same length. Question: change in B.

2. The field depends on the number of turns times the current (per length). 2 × ½ = 1, so the field is unchanged.

3. Answer: B. It stays about the same.

Medium · B10-N06. A coil with an iron core has conventional current going down in the wires on the front of the coil. Which end is the north pole? Why does the iron help?

1. Givens: current down on the front. Question: north pole; role of iron.

2. Grip a front wire with the right hand, thumb down along the current. Inside the coil, behind the wire, the fingers point to the right.

3. The field inside runs right, so field lines leave from the right end: the right end is the north pole.

4. The iron’s domains line up in the coil’s field, adding their own field and making the magnet much stronger.

B10 · Worked practice

Fields around currents | Questions 7-8 of 8

Challenging · B10-N07. A proton moves to the right across the page. At a point above its path, its magnetic field points

A. into the page
B. up the page
C. to the left
D. out of the page

1. Givens: proton moving right. Question: field above the path.

2. A moving proton is a conventional current to the right. Same as a wire with current to the right: above, the field is out of the page.

3. Answer: D. out of the page

Challenging · B10-N08. Using the same moving charge, say what changes if an electron moves to the right instead, and why.

1. Givens: an electron moving right. Question: what changes.

2. An electron moving right is conventional current to the left.

3. The field direction reverses: into the page above the path.

B11 · Magnetic force on a moving charge

Completed notes

Find the size and direction of the magnetic force on a moving charge.

Slide 1 of 5 · What a magnetic force needs

A moving charge carries its own small field. In an outside field the two fields interact and the charge feels a **force**. The force needs a **moving** charge and a **magnetic field**.

Its size is F = **|q|v⊥B**. Here |q| is the **size** of the charge in coulombs, used without its minus sign; v is in **metres per second**; B is in **tesla (T)**, where 1 T = 1 N ÷ (C·m/s). The symbol v⊥ means only the part of the velocity at **right angles** to the field counts.

A charge moving across the field feels the **most** force. A charge moving at an angle feels **less** force. A charge moving along the field lines feels **no** force, and so does a charge that is not moving.

The formula gives only the **size** of the force, so use |q| and decide the direction separately. A **uniform** field has the same size and direction at every point. On the diploma: the data sheet gives the electron's charge, and a numerical response needs **three** significant digits.

Example. An electron moves at 2.5×10^{6} m/s at right angles to a uniform magnetic field of 0.350 T (uniform: the same size and direction at every point). The charge of an electron has size 1.60×10^{-19} C. What is the size of the magnetic force on the electron, in newtons?

1. The electron moves at right angles to the field, so $v_\perp = v = 2.5 \times 10^{6}$ m/s.

2. $F = |q|v_\perp B = (1.60 \times 10^{-19}\ \text{C})(2.5 \times 10^{6}\ \text{m/s})(0.350\ \text{T})$

3. $F = 1.40 \times 10^{-13}$ N.

Try it. A proton (charge 1.60×10^{-19} C) moves at 4.0×10^{6} m/s at right angles to a uniform magnetic field of 0.50 T. What is the size of the magnetic force on it? What would the force be if the proton were at rest?

1. The proton moves at right angles to the field, so $v_\perp = v = 4.0 \times 10^{6}$ m/s.

2. $F = |q|v_\perp B = (1.60 \times 10^{-19}\ \text{C})(4.0 \times 10^{6}\ \text{m/s})(0.50\ \text{T}) = 3.2 \times 10^{-13}$ N.

3. If the proton is at rest, v = 0, so the force is zero. A magnetic force needs a moving charge.

Slide 2 of 5 · Direction: the right-hand rule

Use a flat right hand for this rule (it is a different rule from the curled grip in B10). The magnetic force is at **right angles** to both the velocity and the field. It never points along the **field** or along the **velocity**.

For a positive charge: point your **fingers** along the magnetic field, point your **thumb** along the velocity, and the force pushes out of your **palm**.

Example: the fingers point into the page and the thumb points right, so the palm faces **up** the page and the force on the positive charge is **up** the page.

For a negative charge, find the direction for a **positive** charge first and then **reverse** it. On the diploma: write the positive case first so the reversal is a visible step; a common wrong choice forgets to reverse for an **electron**.

Example. An electron moves to the right through a magnetic field that points out of the page. In which direction is the magnetic force on the electron: up the page, down the page, into the page or out of the page?

1. For a positive charge moving right in a field out of the page, the fingers point at you, the thumb points right and the palm faces down the page, so the force would be down.

2. The electron is negative, so the force is reversed: it points up the page.

Try it. A proton moves straight up the page through a field that points to the left, all in the plane of the page. Which way is the magnetic force on the proton? Which way would it be on an electron moving the same way?

1. The force is at right angles to both vectors, so it cannot be in the plane of the page.

2. Fingers point left (field) and thumb points up (velocity); the palm faces out of the page, so the force on the proton is out of the page.

3. An electron is negative, so the force on it is reversed: into the page.

Slide 3 of 5 · When the motion is at an angle

Split the velocity into a part along the field and a part **across** the field. Only the part across the field, **$v_\perp$**, produces a force.

Draw a right triangle with the velocity as the **long side** (the hypotenuse). Sine = opposite side ÷ **long side**; cosine = side next to the angle ÷ **long side**. Opposite means across from the angle. Set your calculator to **degrees**. If θ is measured between the velocity and the **field**, the part across the field is opposite θ, so $v_\perp = $ **$v \sin \theta$**.

If the angle is measured from the direction **across** the field, that part is next to the angle, so use **cosine**: $v_\perp = v \cos \theta$. Draw the picture and decide which side is across the field before choosing sine or cosine.

Then use $F = $ **$|q|v_\perp B$**. Check that $v_\perp$ is never **bigger** than v. On the diploma: bearing problems use a compass, so **sketch** it; sine versus cosine is a favourite wrong choice.

Example. An electron travels along the ground at 1.8×10^6 m/s in the direction 25° east of south (face south, then turn 25° toward east). The magnetic field is 0.150 T and points east. Use $e = 1.60 \times 10^{-19}$ C. What is the size of the magnetic force on the electron?

1. The field is east. The component of the velocity pointing south is at right angles to it, and it is next to the 25° angle, so $v_\perp = v \cos 25° = (1.8 \times 10^6)(0.90631) = 1.6314 \times 10^6$ m/s.

2. $F = |q|v_\perp B = (1.60 \times 10^{-19})(1.6314 \times 10^6)(0.150) = 3.92 \times 10^{-14}$ N.

Try it. A proton enters a uniform field of 0.50 T moving at 3.0×10^6 m/s at 30° to the field direction. Use $q = 1.60 \times 10^{-19}$ C. What is the size of the force, and what is the force if the proton enters exactly along the field?

1. The angle is measured from the field, so $v_\perp = v \sin 30° = (3.0 \times 10^6)(0.50) = 1.5 \times 10^6$ m/s.

2. $F = |q|v_\perp B = (1.60 \times 10^{-19})(1.5 \times 10^6)(0.50) = 1.2 \times 10^{-13}$ N.

3. Along the field the angle is zero, so $v_\perp = 0$ and the force is zero.

Slide 4 of 5 · Method: how to find the force on a moving charge

Step 1, find **$v_\perp$**: draw the field and the velocity. Only the part of v **across** the field counts. $v_\perp = v$ **sin θ** when θ is measured from the field, and v **cos θ** when it is measured from across the field.

Step 2, find the **size**: $F = |q|v_\perp B$. Use the size of the charge, with no minus sign.

Step 3, find the direction: right hand, **fingers** along the field, **thumb** along the velocity, and the palm gives the force on a positive charge. For a negative charge, **reverse** it.

Step 4, look for **other** forces: if a mass is given, gravity **mg** also acts. “Not deflected” means the magnetic force and gravity **cancel**.

Check: $v_\perp$ can never be **bigger** than v, so the force is never more than **$|q|vB$**. The force is zero along the **field**.

Example. A proton (charge $+1.60 \times 10^{-19}$ C) moves at 4.0×10^{6} m/s at 35° to a magnetic field of 0.620 T. What is the size of the magnetic force on it? (The slide ran through this question.)

1. Step 1: the angle is measured from the field, so $v_\perp = v \sin 35° = (4.0 \times 10^{6})(0.57358) = 2.2943... \times 10^{6}$ m/s. Keep the full value on your calculator.

2. Step 2: $F = |q|v_\perp B = (1.60 \times 10^{-19})(2.2943... \times 10^{6})(0.620) = 2.276... \times 10^{-13}$ N, so $F = 2.28 \times 10^{-13}$ N.

3. Steps 3 and 4: only the size is asked, so no direction is needed, and no mass is given, so there is no gravity to add.

Try it. NEW, for teacher review. An electron moves at 2.0×10^{6} m/s at 40° to a uniform magnetic field of 0.30 T. Use $e = 1.60 \times 10^{-19}$ C. What is the size of the magnetic force on the electron? Write the step number beside each line.

1. Step 1: the angle is measured from the field, so $v_\perp = v \sin 40° = (2.0 \times 10^{6})(0.64279) = 1.2856... \times 10^{6}$ m/s.

2. Step 2: $F = |q|v_\perp B = (1.60 \times 10^{-19})(1.2856... \times 10^{6})(0.30) = 6.171... \times 10^{-14}$ N, so $F = 6.17 \times 10^{-14}$ N.

3. Steps 3 and 4: only the size is asked, and no mass is given. Check: 6.17×10^{-14} N is less than the full $|q|vB = 9.60 \times 10^{-14}$ N, as it must be.

Slide 5 of 5 · Magnetic force against gravity

If a question gives the mass of a particle and asks about forces, **gravity** is part of the story. Gravity on the particle is **mg**, down, with g = 9.81 N/kg (the same as **9.81 m/s²**).

If the particle is not deflected, the net force is **zero**. So the magnetic force equals gravity in size and is opposite in direction: |Fm| = **|Fg|**. Bars mean **size only** and an arrow means size and direction: the two forces have equal size but are not the same vector.

Then |q|vB = **mg**, so v = **mg/(|q|B)**. Write the balance equation as the **first** line of your solution.

When there is only one force, F⃗net = F⃗m is fine. On the diploma: “not deflected” or “straight line” means net force **zero**. If you cannot rearrange, test options with **units**: a speed must end up in m/s.

Example. A particle of mass m and charge q moves horizontally through a magnetic field B that is at right angles to its velocity. It moves in a straight line, because the magnetic force and the force of gravity balance. Which expression gives its speed? A. qB/(mg). B. mg/(qB). C. mgqB. D. qBm/g. Choose one letter.

1. Answer B.

2. A straight path means the net force is zero, so the magnetic force equals the force of gravity in size: qvB = mg.

3. Solving gives v = mg/(qB). The other options do not come from rearranging that equation.

Try it. A particle has mass 6.7×10^{-27} kg and charge $+3.20 \times 10^{-19}$ C and moves horizontally through a horizontal field of 6.0×10^{-5} T at right angles to its path. What speed does it need so that it is not deflected? Use g = 9.81 N/kg.

1. The particle is not deflected, so qvB = mg and v = mg/(qB).

2. v = $(6.7 \times 10^{-27}\ \text{kg})(9.81\ \text{N/kg})/[(3.20 \times 10^{-19}\ \text{C})(6.0 \times 10^{-5}\ \text{T})]$

3. v = 3.42×10^{-3} m/s.

B11 · Worked practice

Magnetic force on a moving charge | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass $m_p = 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B11-N01. An electron moves at 3.6×10^{4} m/s at right angles to a magnetic field of 2.50 T. Find the size of the magnetic force on it. Use $e = 1.60 \times 10^{-19}$ C.

1. Givens: $q = 1.60 \times 10^{-19}$ C; $v = 3.6 \times 10^{4}$ m/s; $B = 2.50$ T; $\theta = 90°$. Question: F.

2. $F = |q|vB = (1.60 \times 10^{-19})(3.6 \times 10^{4})(2.50) = 1.44 \times 10^{-14}$ N.

3. Answer: 1.44×10^{-14} N

Easy · B11-N02. A proton moves toward the top of the page through a field that points into the page. In which direction is the magnetic force?

1. Givens: proton (positive); v up the page; B into the page. Question: force direction.

2. Right hand: fingers along B (into the page), thumb along v of the positive charge (up the page).

3. The palm faces to the left, so the force on the proton is to the left (toward the left side of the page).

B11 · Worked practice

Magnetic force on a moving charge | Questions 3-4 of 8

Easy · B11-N03. Why is the magnetic force on a charged particle zero when it moves along a magnetic field line?

A. Because the particle has no charge
B. Because the field is zero along its own line
C. Because the force is cancelled by gravity
D. Because sin 0° = 0, so no component of v is at right angles to B

1. Givens: v parallel to B. Question: why F = 0.

2. $F = qvB \sin\theta$ and $\theta = 0°$, so F = 0.

3. Answer: D. Because sin 0° = 0, so no component of v is at right angles to B

Medium · B11-N04. A positive charge of 2.0×10^{-6} C moves at 5.0×10^{5} m/s through a field of 0.40 T. The velocity makes an angle of 40° with the field. Find the size of the magnetic force.

1. Givens: $q = 2.0 \times 10^{-6}$ C; $v = 5.0 \times 10^{5}$ m/s; B = 0.40 T; $\theta = 40°$. Question: F.

2. $F = |q|vB \sin\theta = (2.0 \times 10^{-6})(5.0 \times 10^{5})(0.40)(\sin 40°) = 0.257$ N.

3. Answer: 0.26 N

B11 · Worked practice

Magnetic force on a moving charge | Questions 5-6 of 8

Medium · B11-N05. A dust grain with charge 4.0×10^{-9} C and mass 2.0×10^{-7} kg moves horizontally through a horizontal field of 0.50 T at right angles to its path. It is not deflected. What is its speed? Use g = 9.81 N/kg (the same as 9.81 m/s²).

1. Givens: $q = 4.0 \times 10^{-9}$ C; $m = 2.0 \times 10^{-7}$ kg; B = 0.50 T; θ = 90°; g = 9.81 N/kg (= 9.81 m/s²). Question: v.

2. Not deflected means the magnetic force balances gravity: $|q|vB = mg$.

3. $v = mg/(|q|B) = (2.0 \times 10^{-7})(9.81) \div ((4.0 \times 10^{-9})(0.50)) = 981$ m/s.

4. Answer: 981 m/s

Medium · B11-N06. A magnetic field acts on a charged particle at right angles to its velocity. What happens to the particle’s speed?

A. It stays constant, because the force is perpendicular to the velocity.

B. It increases, because there is a force.

C. It decreases, because the field opposes motion.

D. It increases and then decreases.

1. Givens: F □ v. Question: what happens to speed.

2. A force at right angles to the velocity does no work, so the kinetic energy and speed are unchanged; only the direction changes.

3. Answer: A. It stays constant, because the force is perpendicular to the velocity.

B11 · Worked practice

Magnetic force on a moving charge | Questions 7-8 of 8

Challenging · B11-N07. An electron moves at 2.0×10^{6} m/s at right angles to a field of 0.0400 T. Find the size of its acceleration. Use $e = 1.60 \times 10^{-19}$ C and the electron's mass $m_e = 9.11 \times 10^{-31}$ kg.

1. Givens: $q = 1.60 \times 10^{-19}$ C; $v = 2.0 \times 10^{6}$ m/s; $B = 0.0400$ T; $m = 9.11 \times 10^{-31}$ kg. Question: a.

2. $F = |q|vB = (1.60 \times 10^{-19})(2.0 \times 10^{6})(0.0400) = 1.280 \times 10^{-14}$ N. Keep the full value on your calculator.

3. By Newton's second law, $a = F \div m = 1.280 \times 10^{-14} \div (9.11 \times 10^{-31}) = 1.41 \times 10^{16}$ m/s². The acceleration is at right angles to the velocity.

4. Answer: 1.41×10^{16} m/s²

Challenging · B11-N08. A positive and a negative particle with the same speed and charge size move in the same direction through the same field at right angles. Compare the size and direction of their magnetic forces.

1. Givens: same |q|, v and B; opposite signs. Question: compare the forces.

2. $F = |q|vB$ is the same for both.

3. The directions are opposite, because reversing the sign of the charge reverses the force.

B12 · Charged particles on circular paths

Completed notes

Find the radius of a circular path, r = mv/(qB), including after acceleration through a voltage.

Slide 1 of 5 · Why the path is a circle

The magnetic force is at **right angles** to the velocity. No part of the force points along the motion, so it cannot change the **speed**. It only changes the **direction**.

The force always points to one **centre**, so the path is a **circle**. This is uniform **circular** motion. Outside the field the path is **straight**.

The net force toward the centre is the **centripetal** force. Here it is the **magnetic** force. On the diploma: the **speed** stays the same in a magnetic field.

To find which way a particle curves, use the right-hand rule at the first moment (reverse it for an **electron**). The path bends toward the **force**.

Example. An electron moves up the page. A uniform magnetic field covers the whole page and points out of the page, toward you. Does the electron start to curve toward the left side or the right side of the page?

1. For a positive charge moving up through a field out of the page, the right hand gives a force to the right.

2. The electron is negative, so the force is reversed: to the left.

3. The path bends toward the left side of the page.

Try it. An electron enters a region with an electric field between charged plates, and an identical electron enters a magnetic field at the same speed, both moving at right angles to the field. Compare the shape of the two paths, and say what happens to the speed in each.

1. Between the plates the force on the electron always points the same way, so the path is a parabola and the speed increases.

2. In the magnetic field the force is always at right angles to the velocity, so the path is a circle and the speed stays constant.

Slide 2 of 5 · The radius of the circle: r = mv/(qB)

The magnetic force is the net force on the circle. Newton's second law: qvB = **mv²/r**. Divide both sides by v: qB = **mv/r**. Solve for r: r = **mv/(qB)**.

B is the field strength, in **tesla** (T). Put in the **size** of q. The velocity is at **right angles** to B. This formula is **not** on the data sheet, so show where it comes from.

A bigger v or m makes a **bigger** circle. A bigger q or B makes a **smaller** circle.

Use SI units: **kilograms**, metres per second, **coulombs** and tesla. On the diploma: explain a change in the radius using **ratios**.

Example. A proton (mass 1.67×10^{-27} kg, charge 1.60×10^{-19} C) moves at 3.2×10^{5} m/s at right angles to a magnetic field of 0.150 T. What is the radius of its circular path?

1. $qvB = mv^2/r$, so $r = mv/(qB)$.

2. $r = (1.67 \times 10^{-27}\ \text{kg})(3.2 \times 10^{5}\ \text{m/s})/[(1.60 \times 10^{-19}\ \text{C})(0.150\ \text{T})]$

3. $r = 2.23 \times 10^{-2}$ m.

Try it. An electron (mass 9.11×10^{-31} kg, charge size 1.60×10^{-19} C) moves to the right at 6.0×10^{6} m/s at right angles to a field of 2.0×10^{-3} T that points out of the page. What is the radius of its path, and does it circle clockwise or counterclockwise on the page?

1. $r = mv/(qB) = (9.11 \times 10^{-31}\ \text{kg})(6.0 \times 10^{6}\ \text{m/s})/[(1.60 \times 10^{-19}\ \text{C})(2.0 \times 10^{-3}\ \text{T})] = 1.71 \times 10^{-2}$ m.

2. The field is out of the page and the electron moves right. The right hand gives a force down for a positive charge, so the force on the electron is up.

3. The force points toward the centre, so the centre is above the electron and the path turns toward the left: counterclockwise.

Slide 3 of 5 · Accelerate first, then bend

Two stages: a voltage speeds the particle up, then a field bends it. Stage one: the charge gains energy **qΔV** (B6), so $q\Delta V = \mathbf{\frac{1}{2}mv^2}$. This gives the **speed**. ΔV is the voltage, in volts. The particle starts from **rest**.

Stage two: $r = \mathbf{mv/(qB)}$, using the speed from stage one. Use the **size** of q.

Keep **extra** digits in the speed. Use SI units and the **same** charge in both stages. On the diploma: **label** your two stages.

Example. An electron starts at rest and is accelerated through 150 V, then enters a magnetic field of 1.2×10^{-3} T at right angles to its velocity. The electron's mass is 9.11×10^{-31} kg and its charge size is 1.60×10^{-19} C. What is the radius of its path in the magnetic field?

1. Stage one: $q\Delta V = \frac{1}{2}mv^2$, so $v = \sqrt{(2q\Delta V/m)} = \sqrt{[2(1.60 \times 10^{-19})(150)/(9.11 \times 10^{-31})]} = 7.259 \times 10^{6}$ m/s.

2. Stage two: $r = mv/(qB) = (9.11 \times 10^{-31})(7.259 \times 10^{6})/[(1.60 \times 10^{-19})(1.2 \times 10^{-3})]$

3. $r = 3.44 \times 10^{-2}$ m.

Try it. An alpha particle (mass 6.65×10^{-27} kg, charge 3.20×10^{-19} C) starts at rest and is accelerated through 800 V. It then enters a field of 0.0500 T at right angles. What is the radius of its path in the field?

1. Stage one: $v = \sqrt{(2q\Delta V/m)} = \sqrt{[2(3.20 \times 10^{-19})(800)/(6.65 \times 10^{-27})]} = 2.775 \times 10^{5}$ m/s.

2. Stage two: $r = mv/(qB) = (6.65 \times 10^{-27})(2.775 \times 10^{5})/[(3.20 \times 10^{-19})(0.0500)]$

3. $r = 0.115$ m. Use the full alpha charge, 3.20×10^{-19} C, in both stages.

Slide 4 of 5 · Method: how to solve a circular-path problem

Step 1: **split** the problem. Is there an accelerator first? Mark the plates (energy) and the field (**circle**), and name the unknown.

Step 2: find the speed from the energy: $qΔV = \mathbf{½mv^2}$, so $v = \mathbf{\sqrt{(2qΔV/m)}}$, from rest. If the speed is given, **skip** this step. Keep the full value.

Step 3: set the magnetic force equal to the **centripetal** force: $qvB = \mathbf{mv^2/r}$. Solve for the unknown (r, v, m or q) and use the **size** of q.

Step 4: calculate with units: kilograms, **metres per second**, coulombs and tesla. Put in the **full** speed from step 2.

Step 5: find which way it turns. Use the right-hand rule for the first force (reverse it for a **negative** charge). The **centre** is on that side. Check: a faster or heavier particle makes a **bigger** circle, and more charge or field makes a **smaller** one.

Example. An electron starts at rest and is accelerated through 150 V. It then moves to the right into a field of 1.2×10^{-3} T, into the page. Find the radius of its path and which way it turns. ($m = 9.11 \times 10^{-31}$ kg, $q = 1.60 \times 10^{-19}$ C)

1. Step 1: the plates accelerate the electron and the field bends it. The unknowns are r and the turn.

2. Step 2: $v = \sqrt{(2qΔV/m)} = \sqrt{[2(1.60 \times 10^{-19})(150)/(9.11 \times 10^{-31})]} = 7.259\ldots \times 10^{6}$ m/s.

3. Step 3: $qvB = mv^2/r$, so $r = mv/(qB)$. Step 4: $r = (9.11 \times 10^{-31})(7.259 \times 10^{6})/[(1.60 \times 10^{-19})(1.2 \times 10^{-3})] = 3.444 \times 10^{-2}$ m, so $r = 3.44 \times 10^{-2}$ m.

4. Step 5: a positive charge moving right through a field into the page is pushed up. The electron is pushed down, so the centre is below it and the path turns clockwise.

Try it. NEW, for teacher review. A proton starts at rest and is accelerated through 2.00×10^{3} V. It then moves to the right into a field of 0.0400 T, out of the page. Find the radius of its path and which way it turns. ($m = 1.67 \times 10^{-27}$ kg, $q = 1.60 \times 10^{-19}$ C)

1. Step 2: $v = \sqrt{(2qΔV/m)} = \sqrt{[2(1.60 \times 10^{-19})(2.00 \times 10^{3})/(1.67 \times 10^{-27})]} = 6.1906\ldots \times 10^{5}$ m/s.

2. Step 3: $r = mv/(qB)$. Step 4: $r = (1.67 \times 10^{-27})(6.1906 \times 10^{5})/[(1.60 \times 10^{-19})(0.0400)] = 0.1615$ m, so $r = 0.162$ m.

3. Step 5: a positive charge moving right through a field out of the page is pushed down, so the centre is below it and the path turns clockwise.

4. Check: the proton has much more mass than the electron, so its circle is bigger.

Slide 5 of 5 · Using the radius to find the charge

Run the formula backward. Start from qvB = **mv²/r** and solve for the unknown. For the charge, q = **mv/(rB)**. For the field, B = **mv/(qr)**.

Ask what multiplies the unknown. Do the **opposite** to both sides. Check: put the answer back into r = mv/(qB). Is the **radius** the same?

Charge = n × e (B1). Here e = **1.60 × 10⁻¹⁹ C** and n is a whole number of electrons. So n = q ÷ **e**. A negative charge means n **extra** electrons.

The calculator will not give an exact whole number, so write n as a **whole** number. On the diploma: write each rearranging step.

Example. A negatively charged particle of mass 9.6×10^{-27} kg moves at 3.0×10^{5} m/s on a circle of radius 0.0144 m in a field of 0.250 T. What is its charge, and how many extra electrons does it have? ($e = 1.60 \times 10^{-19}$ C)

1. $qvB = mv^2/r$, so $q = mv/(rB)$.

2. $q = (9.6 \times 10^{-27})(3.0 \times 10^{5})/[(0.0144)(0.250)] = 8.00 \times 10^{-19}$ C.

3. Number of electrons = $8.00 \times 10^{-19} \div (1.60 \times 10^{-19}) = 5.00$, so the particle has 5 extra electrons.

Try it. A proton (mass 1.67×10^{-27} kg, charge 1.60×10^{-19} C) moves at 2.4×10^{5} m/s on a circle of radius 0.025 m. What is the strength of the magnetic field?

1. $qvB = mv^2/r$ gives $B = mv/(qr)$.

2. $B = (1.67 \times 10^{-27}\ \text{kg})(2.4 \times 10^{5}\ \text{m/s})/[(1.60 \times 10^{-19}\ \text{C})(0.025\ \text{m})]$

3. B = 0.100 T. Check: r = mv/(qB) gives back 0.025 m.

B12 · Worked practice

Charged particles on circular paths | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}\ N \cdot m^2/C^2$, $e = 1.60 \times 10^{-19}\ C$, electron mass $m_e = 9.11 \times 10^{-31}\ kg$, proton mass $mp = 1.67 \times 10^{-27}\ kg$, $g = 9.81\ m/s^2$.

Easy · B12-N01. Why does a charged particle move in a circle when it enters a magnetic field at right angles to the field?

1. Givens: velocity □ B. Question: why a circle.

2. The magnetic force is perpendicular to the velocity, so it changes only the direction of the velocity.

3. The force has constant size and is always toward the centre, so it is a centripetal force and the path is a circle.

Easy · B12-N02. An electron moves at 2.0×10^{6} m/s at right angles to a field of 0.0400 T. Find the radius of its path. Use $e = 1.60 \times 10^{-19}$ C and the electron's mass $m_e = 9.11 \times 10^{-31}$ kg.

1. Givens: $m = 9.11 \times 10^{-31}$ kg; $v = 2.0 \times 10^{6}$ m/s; $q = 1.60 \times 10^{-19}$ C; $B = 0.0400$ T. Question: r.

2. $r = mv/(qB) = (9.11 \times 10^{-31})(2.0 \times 10^{6}) \div ((1.60 \times 10^{-19})(0.0400)) = 2.85 \times 10^{-4}$ m.

3. Answer: 2.85×10^{-4} m

B12 · Worked practice

Charged particles on circular paths | Questions 3-4 of 8

Easy · B12-N03. A proton moves in a circle in a magnetic field. The field strength is doubled, with the same speed. What happens to the radius?

A. It doubles.
B. It is unchanged.
C. It becomes four times smaller.
D. It is halved.

1. Givens: B doubled; v, m, q unchanged. Question: change in r.

2. r = mv/(qB) so r depends on 1/B. Doubling B halves r.

3. Answer: D. It is halved.

Medium · B12-N04. A proton moving at 1.5×10^{5} m/s enters a field of 0.0800 T at right angles. Find the radius of its circular path. Use the proton's mass mp = 1.67×10^{-27} kg.

1. Givens: m = 1.67×10^{-27} kg; v = 1.5×10^{5} m/s; q = 1.60×10^{-19} C; B = 0.0800 T. Question: r.

2. r = mv/(qB) = $(1.67 \times 10^{-27})(1.5 \times 10^{5}) \div ((1.60 \times 10^{-19})(0.0800))$ = 0.0196 m.

3. Answer: 0.0196 m

B12 · Worked practice

Charged particles on circular paths | Questions 5-6 of 8

Medium · B12-N05. A proton starts at rest and is accelerated through 250 V. It then enters a field of 0.250 T at right angles. Find the radius of its path.

1. Givens: $v_0 = 0$; $\Delta V = 250$ V; $B = 0.250$ T; $q = 1.60 \times 10^{-19}$ C; $m = 1.67 \times 10^{-27}$ kg. Question: r.

2. Step 1, speed: $v = \sqrt{(2q\Delta V/m)} = \sqrt{[2(1.60 \times 10^{-19})(250) \div (1.67 \times 10^{-27})]} = 2.189 \times 10^5$ m/s. Keep the full value on your calculator.

3. Step 2, radius: $r = mv/(qB) = (1.67 \times 10^{-27})(2.189 \times 10^5) \div ((1.60 \times 10^{-19})(0.250)) = 9.14 \times 10^{-3}$ m.

4. Answer: 9.14×10^{-3} m

Medium · B12-N06. An electron and a proton enter the same field at right angles with the same speed. Which has the larger radius?

A. The proton, because it has the larger mass

B. The electron, because it is lighter

C. They have the same radius because the charges are equal in size

D. The electron, because it is negative

1. Givens: same v, B and |q|; different masses. Question: which has the larger r.

2. $r = mv/(qB)$ so r increases with m. The proton is about 1800 times heavier, so its radius is about 1800 times larger.

3. Answer: A. The proton, because it has the larger mass

B12 · Worked practice

Charged particles on circular paths | Questions 7-8 of 8

Challenging · B12-N07. A particle of mass 6.68×10^{-27} kg moves at 2.0×10^{5} m/s at right angles to a field of 0.500 T. Its path has radius 8.35×10^{-3} m. Find the size of its charge.

1. Givens: $m = 6.68 \times 10^{-27}$ kg; $v = 2.0 \times 10^{5}$ m/s; $B = 0.500$ T; $r = 8.35 \times 10^{-3}$ m. Question: q.

2. Rearrange $r = mv/(qB)$ to $q = mv/(rB)$.

3. $q = (6.68 \times 10^{-27})(2.0 \times 10^{5}) \div ((8.35 \times 10^{-3})(0.500)) = 3.20 \times 10^{-19}$ C.

4. That is 2.0 times e, so the charge is 2e.

5. Answer: 3.20×10^{-19} C

Challenging · B12-N08. Compare the path of a charged particle between two parallel charged plates with its path in a magnetic field at right angles to its velocity. Mention the force, the path and the speed.

1. Givens: two situations. Question: compare force, path and speed.

2. Plates: the force $F = qE$ is constant and along the field, so the path is a parabola (if it enters sideways) and the speed changes.

3. Magnetic field: the force is perpendicular to v and constant in size, so the path is a circle and the speed is constant.

B13 · Electric and magnetic fields together

Completed notes

Add the electric and magnetic forces, and use the velocity selector v = E/B.

Slide 1 of 4 · Adding an electric and a magnetic force

Two forces act on the electron. Electric: F = **qE** (charge q, electric field E). Magnetic: F = **qvB** (speed v, magnetic field B).

Top plate +, so E points **down**. A negative charge is pushed **opposite** to E, so the electron is pushed up.

Right hand: up for a positive charge. **Reverse** it for the electron.

Opposite forces: **subtract**. The **larger** force sets the direction. Same way: add.

a = **Fnet ÷ m**. This is the acceleration at the instant of **entry**. Give size and **direction**.

Example. An electron ($e = 1.60 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) moves at 3.50×10^{5} m/s to the right between plates with E = 520 N/C. The top plate is positive. A magnetic field of 2.50×10^{-3} T points into the page. What are the size and direction of the electron’s acceleration as it enters?

1. $F_e = qE = (1.60 \times 10^{-19})(520) = 8.32 \times 10^{-17}$ N. The top plate is positive, so E points down and the force on the electron is up.

2. $F_m = qvB = (1.60 \times 10^{-19})(3.50 \times 10^{5})(2.50 \times 10^{-3}) = 1.40 \times 10^{-16}$ N. The right hand gives up for a positive charge, so it is down for the electron.

3. The forces are opposite: $F_{net} = 1.40 \times 10^{-16} - 8.32 \times 10^{-17} = 5.68 \times 10^{-17}$ N, down (the larger force).

4. $a = F_{net}/m = (5.68 \times 10^{-17})/(9.11 \times 10^{-31}) = 6.23 \times 10^{13}$ m/s², down.

Try it. Same plates and electron, but the magnetic field now points out of the page. What are the size and direction of the electron’s acceleration as it enters?

1. The electric force is unchanged: 8.32×10^{-17} N, up.

2. With B out of the page the magnetic force reverses: it is now up for the electron.

3. Both point up, so they add: $F_{net} = 8.32 \times 10^{-17} + 1.40 \times 10^{-16} = 2.23 \times 10^{-16}$ N, up.

4. $a = (2.23 \times 10^{-16})/(9.11 \times 10^{-31}) = 2.45 \times 10^{14}$ m/s², up. It is larger than before because the forces now add.

Slide 2 of 4 · The velocity selector: v = E/B

A straight path means Fnet = **0**. The forces are equal in size and **opposite** in direction.

The bars | | mean **size only**: |qE| = **|qvB|**.

q is on both sides and **cancels**: E = vB, so v = **E/B**. Mass and charge are not in the equation.

Unit: 1 T = **1 N/(C·m/s)**, so N/C ÷ T is **m/s**. An alpha particle is a helium nucleus, charge **+3.20 × 10⁻¹⁹ C** (twice a proton).

Another speed changes the **magnetic** force but not the **electric** force. The larger force sets the bend.

A positive particle is pushed **with** E, a negative one **against** it. Reverse the right-hand-rule answer for negative charges.

Example. In crossed fields E = 1.20 × 10^{3} N/C (top plate positive, field down) and B = 3.00 × 10^{-3} T (into the page), the straight-through speed is 4.00 × 10^{5} m/s. An electron enters moving right at 5.00 × 10^{5} m/s. Does it bend up or down, and why?

1. The electric force is qE = (1.60 × 10^{-19})(1.20 × 10^{3}) = 1.92 × 10^{-16} N, up. It does not depend on speed.

2. The magnetic force is qvB = (1.60 × 10^{-19})(5.00 × 10^{5})(3.00 × 10^{-3}) = 2.40 × 10^{-16} N, down.

3. The magnetic force is larger, so the electron bends down.

Try it. A proton enters the same fields at 4.00 × 10^{5} m/s (top plate positive, B into the page, moving right). Does it pass straight through? What happens if it is replaced by an alpha particle at the same speed?

1. The field points down, so the electric force on the positive proton is down. The right hand (thumb right, fingers into the page) gives a magnetic force up. The forces are opposite.

2. The sizes cancel because v = E/B = (1.20 × 10^{3})/(3.00 × 10^{-3}) = 4.00 × 10^{5} m/s, so the proton passes straight through.

3. An alpha particle at the same speed also passes straight through: the charge cancels in qE = qvB and the mass does not appear.

Slide 3 of 4 · Method: how to solve a balance-of-forces problem

Step 1: **draw** the particle. One arrow per force: electric, magnetic or **weight**.

Step 2: Electric F = **qE**. Magnetic F = **qvB**. Sizes first. Then directions: plate signs, and the right hand reversed for a **negative** charge.

Step 3: straight path or steady speed: Fnet = **0**. The forces are equal and **opposite**.

Step 4: |qE| = **|qvB|**. The q **cancels**. Solve: v = **E/B**, E = **vB** or B = E/v.

Step 5: check. Arrows **opposite**? Does N/C ÷ T give **m/s**?

Example. An electron goes straight through crossed fields with $E = 1.20 \times 10^{3}$ N/C (top plate positive, field down) and $B = 3.00 \times 10^{-3}$ T (into the page). What is its speed? ($e = 1.60 \times 10^{-19}$ C)

1. Step 1: the electron moves right. E points down, so the electric force is up. The magnetic force is down. The arrows are opposite.

2. Step 2: electric force F = qE. Magnetic force F = qvB, where v is the unknown.

3. Step 3: the path is straight, so Fnet = 0 and the sizes are equal.

4. Step 4: qE = qvB, so $v = E/B = (1.20 \times 10^{3}) \div (3.00 \times 10^{-3}) = 4.00 \times 10^{5}$ m/s.

5. Step 5: $(1.60 \times 10^{-19})(1.20 \times 10^{3}) = 1.92 \times 10^{-16}$ N and $(1.60 \times 10^{-19})(4.00 \times 10^{5})(3.00 \times 10^{-3}) = 1.92 \times 10^{-16}$ N. They match, and N/C ÷ T is m/s.

Try it. NEW, for teacher review. An electron moves right at 5.00×10^{5} m/s and goes straight through plates with $E = 1.00 \times 10^{3}$ N/C (top plate positive, field down). What magnetic field strength and direction are needed? ($e = 1.60 \times 10^{-19}$ C)

1. Step 1: E points down, so the electric force on the electron is up. For the forces to balance, the magnetic force must be down.

2. Step 2 and 3: F = qE and F = qvB, and the path is straight, so Fnet = 0.

3. Step 4: qE = qvB, so $B = E/v = (1.00 \times 10^{3}) \div (5.00 \times 10^{5}) = 2.00 \times 10^{-3}$ T.

4. Direction: fingers into the page and thumb right make the palm face up, which is the force on a positive charge. For the electron the force is down, so B points into the page.

5. Step 5: $(1.60 \times 10^{-19})(1.00 \times 10^{3}) = 1.60 \times 10^{-16}$ N and $(1.60 \times 10^{-19})(5.00 \times 10^{5})(2.00 \times 10^{-3}) = 1.60 \times 10^{-16}$ N. They match.

Slide 4 of 4 · Choosing the tool

Find the **cue** first. Write its equation. Then use the numbers. Do not hunt for a formula that fits.

Straight path: Fnet = **0**. Circle: the **magnetic** force is the net force. Accelerated from rest through ΔV: energy gained is **qΔV**.

Cues: straight **v = E/B**; accelerated from rest **qΔV = ½mv²**; circle **qvB = mv²/r**; two forces **add Fe and Fm as vectors**.

Crossed fields: an **electric** and a **magnetic** field at right angles.

Two cues: one equation per cue, in **order**. Work backward from the **speed** if the end of the path gives it.

$v_\perp$ is the part of v at **right angles** to the field. At rest, or moving **along** the field: $v_\perp = 0$, so F = **0**.

Example. Three cases: (1) an electron is accelerated from rest through 400 V, then moves in a circle in a magnetic field; (2) a proton passes straight through plates and a magnetic field; (3) a positive ion moves along a magnetic field line. For each, write the first equation or statement you would use.

1. Case 1: $q\Delta V = \frac{1}{2}mv^2$ gives the speed, then $qvB = mv^2/r$ gives the radius.

2. Case 2: the path is straight, so the net force is zero and qE = qvB.

3. Case 3: the ion moves along the field, so $v_\perp = 0$ and the magnetic force is zero.

Try it. A proton ($m = 1.67 \times 10^{-27}$ kg) starts at rest, is accelerated through a potential difference, then enters crossed fields $E = 1.20 \times 10^{3}$ N/C and $B = 3.00 \times 10^{-3}$ T and goes straight through. What potential difference accelerated it?

1. Straight path: $v = E/B = (1.20 \times 10^{3})/(3.00 \times 10^{-3}) = 4.00 \times 10^{5}$ m/s.

2. Accelerator: $q\Delta V = \frac{1}{2}mv^2$, so $\Delta V = mv^2/(2q)$.

3. $\Delta V = (1.67 \times 10^{-27})(4.00 \times 10^{5})^2/[2(1.60 \times 10^{-19})] = 835$ V.

B13 · Worked practice

Electric and magnetic fields together | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass $m_p = 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B13-N01. An electron moves at 4.0×10^{5} m/s through a magnetic field of 3.0×10^{-3} T and an electric field of 600 N/C. The two forces are in opposite directions. Find the size of the net force on the electron.

1. Givens: $q = 1.60 \times 10^{-19}$ C; $v = 4.0 \times 10^{5}$ m/s; $B = 3.0 \times 10^{-3}$ T; $E = 600$ N/C; forces opposite. Question: Fnet.

2. Magnetic: $qvB = 1.92 \times 10^{-16}$ N. Electric: $qE = 9.60 \times 10^{-17}$ N.

3. Subtract (opposite directions): 9.60×10^{-17} N, in the direction of the larger force, the magnetic one.

4. Answer: 9.60×10^{-17} N

Easy · B13-N02. A proton moves at 2.0×10^{5} m/s through a magnetic field of 0.0400 T. What electric field strength makes it go straight through? (N/C)

1. Givens: $v = 2.0 \times 10^{5}$ m/s; $B = 0.0400$ T. Question: E.

2. Straight through means $qE = qvB$, so $E = vB$.

3. $E = (2.0 \times 10^{5})(0.0400) = 8000$ N/C.

4. Answer: 8000 N/C

B13 · Worked practice

Electric and magnetic fields together | Questions 3-4 of 8

Easy · B13-N03. Does the speed that passes straight through a velocity selector change if the particle is an alpha particle (twice the charge of a proton) instead of a proton?

A. Yes, it doubles because the charge doubles
B. Yes, it halves because the mass is larger
C. Yes, it depends on the sign only
D. No, because v = E/B does not depend on the charge or mass

1. Givens: same fields; double charge. Question: does the selected speed change.

2. The charge cancels in qE = qvB. The selected speed is v = E/B for any charge or mass.

3. Answer: D. No, because v = E/B does not depend on the charge or mass

Medium · B13-N04. Fields E = 600 N/C and B = 1.50×10^{-3} T are at right angles in a velocity selector. What speed passes straight through?

1. Givens: E = 600 N/C; B = 1.50×10^{-3} T. Question: v.

2. v = E/B = 600 ÷ (1.50×10^{-3}) = 4.00×10^{5} m/s.

3. Answer: 4.00×10^{5} m/s

B13 · Worked practice

Electric and magnetic fields together | Questions 5-6 of 8

Medium · B13-N05. In a velocity selector the top plate is positive and the magnetic field points into the page. A proton moves right, slightly slower than E/B. Which way does it bend?

A. Up, toward the positive plate
B. Down, toward the negative plate
C. Into the page
D. Out of the page

1. Givens: top plate +; B into page; proton moving right with v < E/B. Question: direction it bends.

2. Electric force on a proton is from + to −: down. Magnetic force on a proton moving right in a field into the page is up (right-hand rule).

3. The magnetic force is proportional to v. A slower proton has a smaller magnetic force, so the downward electric force is larger: it bends down.

4. Answer: B. Down, toward the negative plate

Medium · B13-N06. Using the electron in the first question (net force 9.60×10^{-17} N), find the size of its acceleration at the instant it enters the fields. Use $m_e = 9.11 \times 10^{-31}$ kg.

1. Givens: $F_{net} = 9.60 \times 10^{-17}$ N; $m = 9.11 \times 10^{-31}$ kg. Question: a at entry.

2. $a = F/m = 1.05 \times 10^{14}$ m/s².

3. Answer: 1.05×10^{14} m/s²

B13 · Worked practice

Electric and magnetic fields together | Questions 7-8 of 8

Challenging · B13-N07. A velocity selector must pass particles at 2.5×10^5 m/s through a magnetic field of 0.0300 T. The electric field comes from plates 0.0200 m apart. What potential difference is needed?

1. Givens: $v = 2.5 \times 10^5$ m/s; $B = 0.0300$ T; $\Delta d = 0.0200$ m (the distance between the plates). Question: ΔV.

2. Straight through: $E = vB = 7500$ V/m.

3. $\Delta V = E\Delta d = (7500)(0.0200) = 150$ V.

4. Answer: 150 V

Challenging · B13-N08. Why do problems with both an electric and a magnetic field usually ask only for the acceleration at the instant of entry?

1. Givens: both fields act. Question: why the instant of entry.

2. The magnetic force depends on the velocity, and the velocity changes direction and size as the particle moves.

3. So the net force and acceleration keep changing; at the instant of entry the velocity is known, so we can find them.

B14 · The motor effect

Completed notes

Find the direction and size of the magnetic force on a current-carrying wire, F = ILB.

Slide 1 of 7 · Why a wire in a field is pushed

A wire carries a huge number of moving charges, all moving the **same** way, so each feels the force in the same direction and the whole wire is pushed. This is the motor effect: **current in, force out**.

Conventional current is the way **positive** charge would move, so it points **opposite** to the electron flow. Every hand rule in this course uses conventional current.

Right-hand rule for a wire (the same rule as for one charge in B11): fingers along the **magnetic field**, thumb along the **conventional current**, **palm** shows the force. For electron flow, point the thumb along the **electrons** and the force comes out of the **back of the hand**.

Example: fingers into the page, thumb right, palm faces **up**, so the force on the wire is up the page. Reversing the field or the current flips the force; reversing both flips it **twice**.

Example. A wire carries conventional current from left to right through a magnetic field that points out of the page. In which direction is the magnetic force on the wire?

1. Fingers point out of the page (along B) and the thumb points right (along the current).

2. The palm faces down the page, so the force on the wire is down the page.

3. This is opposite to the case with the field into the page, as expected when the field is reversed.

Try it. A vertical wire in the plane of the page carries electrons moving down the page. The magnetic field points to the right across the page. Which way is the force on the wire: into the page, out of the page, left or right?

1. Electrons move down, so conventional current is up the page.

2. Fingers point right (along B) and the thumb points up (along the current).

3. The palm faces into the page, so the force on the wire is into the page.

Slide 2 of 7 · The size of the force: F = I L B

More current, more length in the field and a stronger field each give more force. They multiply: F = **I L⊥ B**, where L⊥ is the length **across the field**. Only the length **inside the field** counts.

Current is charge per time: I = **q ÷ t**, in amperes, where one ampere is **one coulomb per second**. Units: 1 T = **1 N/(A·m)**, so A × m × T gives **newtons**.

θ is the angle between the **wire** and the **field**, and L⊥ = **L sin θ**. As for velocity in B11, only the part across the field counts. At 90°, L⊥ = **L**.

Numerical questions often hide a step, such as finding I from **q and t**. Check that the angle is measured from the **field**, then decide sine or cosine. A picture settles it.

Example. 18 C of charge pass a point in a wire in 4.0 s. A 0.120 m length of the wire lies at right angles to a uniform magnetic field of 0.450 T. What is the current, and what is the size of the magnetic force on the wire?

1. I = q/t = 18 C ÷ 4.0 s = 4.5 A.

2. F = ILB = (4.5 A)(0.120 m)(0.450 T) = 0.243 N.

Try it. A straight wire of length 0.20 m carries 6.0 A through a magnetic field of 0.500 T, at 30° to the field direction. What is the size of the magnetic force on the wire?

1. The part of the length across the field is L sin 30° = (0.20 m)(0.500) = 0.10 m.

2. F = I(L sin θ)B = (6.0 A)(0.10 m)(0.500 T) = 0.300 N.

3. Check: at 90° the force would be 0.600 N, so 0.300 N at 30° is smaller, as it should be.

Slide 3 of 7 · Magnetic force against gravity

A wire in a field can feel the magnetic force and gravity at once. Gravity is **mg** downward; convert the mass from **grams** to kilograms. g = 9.81 **N/kg**, the same as 9.81 m/s².

If the two forces point opposite ways, **subtract** the sizes; the net force points the way of the larger force.

A wire that hangs still has net force **zero**, so ILB = **mg** and I = **mg ÷ (L B)**. “Suspended” and “hangs motionless” both mean net force zero. Size check: a weaker field needs a **larger** current.

Equal in size and opposite in direction is not the same as equal as **vectors**. A stationary wire can still have forces on it; they just **balance**.

Example. A horizontal wire of mass 1.40 g has 0.0900 m of its length in a horizontal magnetic field of 0.0500 T. The north pole is on the left, the south pole on the right, and the current is 4.00 A toward you. What are the magnetic force, the weight, and the net force on the wire?

1. Field points north to south, to the right. Fingers right, thumb toward you, palm faces up: the magnetic force is up.

2. Fm = ILB = (4.00)(0.0900)(0.0500) = 0.01800 N, up. Weight = mg = $(1.40 \times 10^{-3})(9.81)$ = 0.01373 N, down.

3. Forces are opposite: Fnet = 0.01800 − 0.01373 = 4.27×10^{-3} N, up.

Try it. The same wire (1.40 g, 0.0900 m in a 0.0500 T field) hangs still between the magnets with the current toward you. What current is needed to suspend it? Use g = 9.81 N/kg.

1. A wire at rest has Fnet = 0, so ILB = mg.

2. I = mg/(LB) = $(1.40 \times 10^{-3}$ kg)(9.81 N/kg)/[(0.0900 m)(0.0500 T)] = 3.05 A.

Slide 4 of 7 · Method: how to choose the tool for a wire in a field

Step 1: write what is **asked** and every **given**. Note if the wire is at **rest**.

Step 2: a current in a wire uses F = **ILB**. One moving charge uses F = **qvB**, from lesson B11.

Step 3: use only the length **in the field**, and L sin θ if the wire is **slanted**. Change grams to **kilograms**.

Step 4: right hand: fingers along the **field**, thumb along **conventional** current, palm shows the **force**. Electron flow? **Reverse** it first.

Step 5: gravity pulls **down** with mg. A wire at rest has Fnet = **0**, so ILB = **mg**. Otherwise subtract the smaller force from the larger.

Check: units A·m·T give **N**. Do the two forces point **opposite** ways? On the diploma: a wire at rest is **balanced**, not “without force”.

Example. A horizontal wire of mass 1.40 g hangs still between a north pole on the left and a south pole on the right. 0.0900 m of its length is in the field, the field is 0.0500 T and the current is toward you. What current holds the wire still, and which way is the magnetic force?

1. Step 1: asked I. Given m = 1.40 g, L = 0.0900 m, B = 0.0500 T. The wire is at rest.

2. Step 2: a current in a wire, so F = ILB, not qvB.

3. Step 3: the wire is at right angles to the field, so L = 0.0900 m. Change the mass: 1.40 g = 1.40×10^{-3} kg.

4. Step 4: the field points right, north to south. Fingers right, thumb toward you, palm up: the magnetic force is up and gravity is down.

5. Step 5: at rest, so Fnet = 0 and ILB = mg. mg = $(1.40 \times 10^{-3})(9.81) = 1.3734 \times 10^{-2}$ N. I = $1.3734 \times 10^{-2} \div [(0.0900)(0.0500)] = 3.052$ A, so I = 3.05 A.

Try it. NEW, for teacher review. A horizontal wire of mass 2.40 g has 0.150 m of its length in a horizontal field of 0.0800 T. The north pole is on the right, the south pole is on the left and the current is away from you. The wire hangs still. What is the current, and is the magnetic force up or down?

1. Step 1: asked I. Given m = 2.40 g, L = 0.150 m, B = 0.0800 T. The wire is at rest.

2. Steps 2 and 3: a current in a wire, so F = ILB. L = 0.150 m and 2.40 g = 2.40×10^{-3} kg.

3. Step 4: the field points left, north to south. Fingers left, thumb away from you, palm up: the magnetic force is up.

4. Step 5: at rest, so ILB = mg. mg = $(2.40 \times 10^{-3})(9.81) = 2.3544 \times 10^{-2}$ N.

5. I = $2.3544 \times 10^{-2} \div [(0.150)(0.0800)] = 1.962$ A, so I = 1.96 A.

Slide 5 of 7 · Reading a force graph

A wire rests on a balance and carries a current in a magnet’s field, so the magnetic force pushes **down**. The balance reads F = **I L B + mg**. Plot F against I and match it to y = (slope)x + (intercept), as in B5.

The slope is **L B** and the vertical intercept is **mg**, the force of gravity when the current is zero. The line does not start at zero because gravity always pulls **down**.

From the line: B = **slope ÷ L** and m = **intercept ÷ g**. Use kilograms for mass and tesla for field.

Pick two points **far apart**, not data points. Show the slope and intercept steps with **units**. To predict a reading use the equation of the line, because ILB alone leaves out the weight.

Example. A balance reading F against current I gives a straight line through P (0.80 A, 0.530 N) and Q (4.00 A, 0.786 N). The length of wire in the field is 0.120 m. Find the field strength B and the mass of the wire and supports.

1. Slope = (0.786 − 0.530)/(4.00 − 0.80) = 0.0800 N per A = LB, so B = 0.0800/0.120 = 0.667 T.

2. Intercept = 0.530 − (0.0800)(0.80) = 0.466 N = mg.

3. m = 0.466/9.81 = 4.75×10^{-2} kg.

Try it. Using the same line, F = (0.0800 N/A)I + 0.466 N, predict the balance reading when the current is 2.50 A.

1. Use the equation of the line, not ILB alone, because the weight is included.

2. F = (0.0800)(2.50) + 0.466 = 0.200 + 0.466 = 0.666 N.

3. Check: it lies between 0.530 N (0.80 A) and 0.786 N (4.00 A), so it is reasonable.

Slide 6 of 7 · Two wires, two forces

Each wire makes a field, and each sits in the other’s field. Do two steps: find the **field** of one wire at the other, then the **force** on the second wire.

Rule 1 (the field of a wire): grip with your right hand, **thumb** along the current; the fingers curl the way the field goes. For current up, the field on the right side of the wire is **into** the page.

Rule 2 (the force on a wire): fingers along the **field**, thumb along the **current**, **palm** shows the force.

Currents in the **same** direction attract; currents in **opposite** directions repel. The two forces are equal in size and opposite in **direction**.

Use the same-direction rule as a **check** on your answer. A slinky with current shrinks because neighbouring turns **attract**.

Example. Wire A carries conventional current up the page. Wire B, to its right, carries conventional current down the page. Do the wires attract or repel, and which way is the force on B?

1. The field of A at B is into the page (thumb up, right side).

2. B’s current is down, so the thumb points down and the fingers point into the page: the palm faces right.

3. The force on B is to the right, away from A, so currents in opposite directions repel.

Try it. Wire A carries conventional current down the page. Wire B, to its right, also carries conventional current down the page. Which way is the force on B, and do the wires attract or repel?

1. Thumb down on A: the field on the right side of A, at B, comes out of the page.

2. B’s current is down (thumb down), fingers out of the page: the palm faces left.

3. The force on B is left, toward A, so the wires attract (same-direction currents attract).

Slide 7 of 7 · How a motor’s coil turns

A motor is a **loop** of wire in a magnetic field. Each side of the loop is a wire in a field, so use the same rule: fingers along the **field**, thumb along the **current**, palm shows the force.

The two sides carry current in **opposite** directions, so the forces on them are **opposite**. One side is pushed into the page and the other out of the page, so the loop **turns**.

The top and bottom sides run **along** the field, so they feel **no** force.

After half a turn the sides swap places. The **commutator** (a split ring) reverses the **current** every half turn, so the loop keeps turning the **same** way.

More **current** or more **turns** of wire gives a larger force. Reverse the current, or the field, and the motor turns the **opposite** way.

Example. A rectangular loop sits between a north pole on the left and a south pole on the right, so the field points to the right. Conventional current goes down the left side of the loop and up the right side. (a) Which way is the force on the left side, and on the right side? (b) Does the loop turn the same way as it does with current up the left side, or the opposite way?

1. (a) Left side: fingers right, thumb down, so the palm faces out of the page. Right side: thumb up, so the palm faces into the page.

2. (b) Both forces have reversed, so the loop turns the opposite way. Reversing the current reverses a motor.

Try it. NEW, for teacher review: The same loop has conventional current up its left side and down its right side. The magnets are swapped, so the field now points to the left. Which way is the force on the left side, and on the right side?

1. Left side: fingers left, thumb up, so the palm faces out of the page.

2. Right side: fingers left, thumb down, so the palm faces into the page.

3. Reversing the field reverses both forces, so the motor turns the opposite way.

B14 · Worked practice

The motor effect | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass $m_p = 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B14-N01. A wire carries conventional current to the right through a magnetic field that points up the page. The magnetic force on the wire is

A. into the page
B. out of the page
C. up the page
D. to the right

1. Givens: I to the right; B up the page. Question: direction of the force.

2. Right hand: fingers up the page (along B), thumb right (along I). The palm faces out of the page.

3. Answer: B. out of the page

Easy · B14-N02. A 0.250 m length of wire carries 5.0 A at right angles to a magnetic field of 0.80 T. Find the magnetic force on the wire.

1. Givens: L = 0.250 m; I = 5.0 A; B = 0.80 T. Question: F.

2. F = ILB = (5.0)(0.250)(0.80) = 1.0 N.

3. Answer: 1.00 N

B14 · Worked practice

The motor effect | Questions 3-4 of 8

Easy · B14-N03. 12 C of charge pass through a wire in 3.0 s. A 0.100 m length of the wire is at right angles to a field of 0.50 T. Find the magnetic force on that length.

1. Givens: q = 12 C; t = 3.0 s; L = 0.100 m; B = 0.50 T. Question: F.

2. I = q/t = 12 ÷ 3.0 = 4.0 A.

3. F = ILB = (4.0)(0.100)(0.50) = 0.20 N.

4. Answer: 0.200 N

Medium · B14-N04. A wire of mass 2.4 g has 0.100 m of its length at right angles to a field of 0.250 T. What current makes the magnetic force exactly balance gravity? Use g = 9.81 m/s².

1. Givens: m = 2.4 × 10^{-3} kg; L = 0.100 m; B = 0.250 T; g = 9.81 m/s². Question: I.

2. Balance: ILB = mg.

3. I = mg/(LB) = (2.4 × 10^{-3})(9.81) ÷ ((0.100)(0.250)) = 0.94 A.

4. Answer: 0.942 A

B14 · Worked practice

Medium · B14-N05. A wire of mass 1.40 g has 0.0900 m of its length in a field of 0.0500 T. A current of 4.00 A flows so that the magnetic force points down. Find the size of the net force on the wire.

1. Givens: m = 1.40 × 10^{-3} kg; L = 0.0900 m; B = 0.0500 T; I = 4.00 A; magnetic force down. Question: Fnet.

2. Magnetic: ILB = 0.0180 N down. Gravity: mg = 0.0137 N down.

3. Both point down, so add: 0.0317 N downward.

4. Answer: 0.0317 N

Medium · B14-N06. A graph of scale reading against current for a wire in a field gives the line F = 0.0800 I + 0.466 (F in newtons, I in amperes). What should the scale read at 5.5 A?

1. Givens: F = 0.0800 I + 0.466; I = 5.5 A. Question: F.

2. F = 0.0800(5.5) + 0.466 = 0.906 N.

3. 5.5 A should be within the measured current range for the prediction to be reliable.

4. Answer: 0.906 N

B14 · Worked practice

The motor effect | Questions 7-8 of 8

Challenging · B14-N07. Two parallel wires carry currents in the same direction. Do they attract or repel? Explain using the field of one wire and the force on the other.

1. Givens: parallel wires, same direction. Question: attract or repel, and why.

2. Wire 1 makes a magnetic field at wire 2 (right-hand rule, thumb along the current).

3. The current in wire 2 is at right angles to that field, so by the right-hand rule the force on wire 2 points toward wire 1. The same argument gives the force on wire 1 toward wire 2: they attract.

Challenging · B14-N08. A horizontal wire of mass 5.00 g has 0.200 m of its length in a horizontal field of 0.150 T at right angles to the wire. What current holds the wire up, and what direction must the force have? Give the current in A. Use g = 9.81 m/s².

1. Givens: $m = 5.00 \times 10^{-3}$ kg; L = 0.200 m; B = 0.150 T; g = 9.81 m/s². Question: I (and the force direction).

2. The magnetic force must be upward and equal in size to the weight: ILB = mg.

3. $I = mg/(LB) = (5.00 \times 10^{-3})(9.81) \div ((0.200)(0.150)) = 1.64$ A.

4. Answer: 1.64 A

B15 · The generator effect and Lenz’s law

Completed notes

Explain how motion in a field separates charge, use ΔV = Blv, and apply Lenz’s law.

Slide 1 of 8 · A moving rod separates charge

The generator effect runs the motor effect backward: **motion in, electricity out**. Pulling a rod through a field pulls its **electrons** through the field too. Each feels a force.

Right hand: fingers along the **field**, thumb along the rod’s **velocity** (in B14 the thumb pointed along the current). The palm shows the force on a **positive** charge. **Reverse** it for electrons.

The electrons gather at one end, so that end is **negative**. The other end is **positive**.

Reverse the velocity or the field, and the polarity **flips**.

Example. A horizontal metal rod is pulled straight down the page through a magnetic field that points into the page. Which end of the rod, left or right, becomes positive?

1. For a positive charge: thumb down (velocity), fingers into the page (field), so the force is to the right.

2. Electrons are negative, so they are pushed to the left and gather at the left end.

3. The right end is positive.

Try it. A vertical rod is pulled to the right through a magnetic field that points out of the page. Which end, top or bottom, becomes positive, and how does this compare with the field into the page?

1. Fingers out of the page, thumb right: the palm faces down, so a positive charge is pushed down.

2. The electrons are pushed up, so the top end is negative and the bottom end is positive.

3. This is the opposite of the case with the field into the page.

Slide 2 of 8 · How large is the potential difference?

ΔV is the potential difference between the **two ends** of the rod. ΔV = **Blv**. B is the field (T), l is the rod length (m), v is the speed (m/s).

Faster rod, longer rod or stronger field: a **bigger** ΔV. 1 T × 1 m × 1 m/s = **1 V**. Example: (0.250 T)(0.40 m)(3.0 m/s) = **0.300 V**.

The equation is **not** on the data sheet. Write the rearranged form, such as v = **ΔV/(Bl)**, before the numbers.

For ratio questions, **multiply** the change factors. Never add them.

Example. A rod of length 0.60 m moves at right angles to a magnetic field of 0.150 T and to its own length. The potential difference between its ends is 0.90 V. What is the speed of the rod? (ΔV = Blv)

1. ΔV = Blv, so v = ΔV/(Bl).

2. v = 0.90 V ÷ [(0.150 T)(0.60 m)] = 10.0 m/s.

Try it. A rod moving through a field produces a potential difference ΔV. A second rod has twice the length, moves at half the speed, and is in a field three times as strong. What is its potential difference? A. ΔV/3 B. ΔV C. 3ΔV D. 6ΔV

1. ΔV is proportional to Blv. The factors are 2 for the length, ½ for the speed and 3 for the field.

2. 2 × ½ × 3 = 3, so the new potential difference is 3ΔV.

3. Answer C.

Slide 3 of 8 · Closing the circuit

The rod slides on two **rails** (parallel metal bars) with a **resistor** (a part that uses electrical energy, like a lamp) joining the ends. The rod acts like a **battery**.

Conventional current leaves the **positive** end, goes round the circuit and returns to the **negative** end. Inside the rod it goes from negative to positive.

Find the ends first (right hand, then reverse). Then **follow** the current: clockwise or counterclockwise.

Reverse the velocity or the field, and the current **reverses**.

Example. A rod slides on two rails. A resistor joins the right-hand ends of the rails, and the magnetic field points into the page. When the rod is pulled to the right the top end is positive and the current is clockwise. Now the rod is pulled to the left. Does conventional current go clockwise or counterclockwise?

1. With the rod moving left, the force on the electrons is up, so they gather at the top: the bottom end is positive.

2. Conventional current leaves the bottom end and goes the opposite way round from the earlier case: counterclockwise.

Try it. Same rails and rod, but the field now points out of the page and the rod is pulled to the right. Does conventional current go clockwise or counterclockwise, and which end of the rod is positive?

1. Fingers out of the page, thumb right: the palm faces down, so a positive charge is pushed down and the electrons are pushed up.

2. The bottom end is positive.

3. Conventional current leaves the bottom end and flows counterclockwise.

Slide 4 of 8 · A magnet and a coil

A coil can do it too. When a magnet and a coil move **relative** to each other, the electrons in the coil feel a force. This is electromagnetic **induction**. Michael **Faraday** discovered it in 1831.

Current flows only **while** they move. A magnet held still gives **zero** current.

Reverse the motion or the pole, and the current **reverses**.

Faster, a stronger magnet or more turns gives **more** current. Four times the turns gives about **four times** the current.

Example. Pushing the south pole of a bar magnet into a coil at a steady speed deflects the meter needle to the right. Predict the needle for (a) pulling the same pole out at the same speed, (b) pushing the north pole in at the same speed, (c) pushing the south pole in twice as fast, and (d) holding the magnet still inside the coil.

1. (a) Pulling out reverses the motion, so the needle goes left.

2. (b) Pushing the north pole in reverses the pole, so the needle goes left.

3. (c) Twice as fast: same direction (right) with a larger deflection, about twice.

4. (d) A magnet held still gives no current, so the needle stays at zero.

Try it. Pushing the south pole into a coil at a steady speed gives a deflection of 2 divisions to the right. Predict the needle if the same pole is pushed in at half the speed into a coil with four times as many turns.

1. Same pole and same direction of motion, so the needle goes right.

2. Half the speed gives half the size; four times the turns gives four times the size: 4 × ½ = 2 times.

3. The deflection is about 4 divisions to the right.

Slide 5 of 8 · Motors and generators

A motor and a generator are the same device used both ways. Motor: electricity in, **motion** out. Generator: motion in, **electricity** out.

Name the energy change: motion to **electricity**, or the reverse.

A generator joined to a lamp is **harder** to turn. Its current is in a field, so it feels a force that **resists** the turning.

With nothing connected there is no **current**, so no force. The extra **work** you do goes into the lamp’s energy.

Example. A hand-cranked generator is easy to turn when its wires are not connected to anything, and noticeably harder when it is connected to a lamp that lights. Explain why it is harder when connected, and where the extra energy goes.

1. With the circuit open there is no current, so there is no magnetic force on the coil and it turns easily.

2. With the lamp connected there is a current in a magnetic field, so there is a magnetic force that resists the turning.

3. The extra work you do against that force becomes the lamp’s energy.

Try it. A small toy motor is spun by hand with its two wires joined to a lamp, and the lamp lights. Is the device acting as a motor or a generator? Name the energy change.

1. It is acting as a generator: the coil is turned in a field, so the generator effect gives a voltage and a current.

2. Energy change: motion (kinetic energy from your hand) to electrical energy, which the lamp changes to light and heat.

Slide 6 of 8 · Lenz’s law: the force opposes the motion

A moving magnet makes currents in a conductor. A current made by motion in a field, with no battery, is an **induced** current. It feels a **force**, and the force acts on the magnet.

Lenz’s law: the force from an induced current **opposes** the motion that caused it. A magnet falling feels a force **up**.

Chain: motion → **induced current** → **opposing force** → slower fall. No current means **no** force. A better conductor carries a **larger** current.

The force always opposes the motion. It takes energy **away**.

In air the magnet falls 0.30 m in **0.25 s**. In the aluminium tube it takes **2–3 s**.

Example. Three rods of the same size, one plastic, one aluminium and one copper, each have a ring magnet dropped over them from the same height. Copper conducts better than aluminium; plastic is an insulator. Order the rods from fastest fall to slowest and explain.

1. Plastic is fastest. It is an insulator, so there is no induced current, no force and the magnet falls as if the rod were not there.

2. Aluminium is slower: it carries induced currents that give an opposing force.

3. Copper is slowest: it is the better conductor, so the current and the opposing force are largest.

Try it. A pendulum with a strong magnet at its end swings between two aluminium blocks and is moving to the right through its lowest point. What is the direction of the magnetic force on the magnet right now, what happens to the swing over time, and does this force’s direction change when it swings back to the left?

1. Moving right, the force on the magnet is to the left, opposing the motion.

2. The swing dies away quickly, because the force always opposes the motion and removes energy.

3. When it swings left the force is to the right: the direction changes with the motion, but it always opposes it.

Slide 7 of 8 · Lenz’s law for a coil: which way the current goes

An induced current turns a coil into a **magnet**. By Lenz’s law, that magnet **opposes** the motion that caused the current.

Step 1: which way is the magnet **moving**? Step 2: which pole opposes that? A magnet moving closer must be **repelled**, so the near end is the **same** pole. A magnet moving away must be **attracted**, so the near end is the **opposite** pole.

Step 3: the field inside a coil points toward its **north** end. Grip a wire on the **front** of the coil with the right hand, thumb along the conventional current. The fingers inside the coil show the field.

Thumb up puts the field inside to the **left**. Thumb down puts it to the **right**. Reverse the motion and the pole and the current both **reverse**.

Example. A coil is joined to a meter in a closed circuit. The north pole of a bar magnet is pushed toward the right end of the coil. (a) Which pole does the right end of the coil become? (b) Which way does the conventional current go in the wires on the front of the coil, up or down? (c) The magnet is then pulled back out. What happens to the pole and to the current?

1. (a) The magnet is moving closer, so the coil must repel it. The right end becomes a north pole.

2. (b) The field inside the coil points right, toward the north end. Thumb down on a front wire puts the fingers pointing right inside the coil, so the current in the front wires is down.

3. (c) The coil must now attract the magnet, so the right end becomes a south pole and the current reverses to up.

Try it. NEW, for teacher review: The south pole of a bar magnet is pushed toward the left end of a coil in a closed circuit. Which pole does the left end of the coil become, and which way does the conventional current go in the wires on the front of the coil?

1. The magnet is moving closer, so the coil must repel it. The left end becomes a south pole, so the right end is the north pole.

2. The field inside the coil points right, toward the north end.

3. Thumb down on a front wire puts the fingers pointing right inside the coil, so the current in the front wires is down.

Slide 8 of 8 · Method: how to find an induced voltage and its direction

Step 1: list v, B and l with units: **m/s**, **T**, **m**. All at **right angles**.

Step 2: ΔV = **Blv**. Keep every digit. The unit is **volts**.

Step 3: right hand, then reverse for electrons. They gather at the **negative** end. The other end is **positive**.

Step 4: current leaves the **positive** end. Step 5: **Lenz's law**: the force on the rod **opposes** v. Reverse v or B, and every direction **flips**.

Example. A 0.40 m rod is pulled to the right at 3.0 m/s through a field of 0.250 T that points into the page. The rod is part of a closed circuit with a resistor on the right. Find the potential difference, the positive end, the direction of the current and the direction of the force on the rod.

1. Step 1: v = 3.0 m/s to the right, B = 0.250 T into the page, l = 0.40 m.

2. Step 2: ΔV = Blv = (0.250)(0.40)(3.0) = 0.300 V.

3. Step 3: fingers into the page, thumb right: the palm faces up, so the electrons are pushed down. The bottom end is negative and the top end is positive.

4. Step 4: current leaves the top end, goes along the top rail, down through the resistor and back along the bottom rail: clockwise.

5. Step 5: in the rod the current goes up, so the palm faces left. The force on the rod is to the left, opposite to v. Lenz's law agrees.

Try it. A 0.60 m rod is pulled to the left at 2.0 m/s through a field of 0.500 T that points into the page. The rod is part of a closed circuit with a resistor on the right. Find the potential difference, the positive end, the direction of the current and the direction of the force on the rod. Write each step number beside your work.

1. Step 1: v = 2.0 m/s to the left, B = 0.500 T into the page, l = 0.60 m.

2. Step 2: ΔV = Blv = (0.500)(0.60)(2.0) = 0.600 V.

3. Step 3: fingers into the page, thumb left: the palm faces down, so the electrons are pushed up. The top end is negative and the bottom end is positive.

4. Step 4: current leaves the bottom end, goes right along the bottom rail, up through the resistor and back along the top rail: counterclockwise.

5. Step 5: in the rod the current goes down, so the palm faces right. The force on the rod is to the right, opposite to v. Lenz's law agrees.

B15 · Worked practice

The generator effect and Lenz’s law | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass $m_p = 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B15-N01. What is the difference between the motor effect and the generator effect?

1. Givens: none. Question: the difference.

2. Motor effect: a current in a magnetic field gives a force (electrical energy into motion).

3. Generator effect: motion in a magnetic field gives a potential difference (motion into electrical energy).

Easy · B15-N02. A vertical rod is pulled to the right through a magnetic field that points into the page. Which end of the rod becomes positive?

A. The top end
B. The bottom end
C. The left end
D. Neither end

1. Givens: rod vertical; v right; B into the page. Question: which end is positive.

2. Treat the charges in the rod as moving right in the field. For a positive charge moving right in a field into the page, the force is up (right-hand rule from B11).

3. Positive charge is pushed to the top, so the top end is positive.

4. Answer: A. The top end

B15 · Worked practice

Easy · B15-N03. A 0.80 m rod moves at 5.0 m/s at right angles to a field of 0.040 T and to its own length. Find the potential difference between its ends.

1. Givens: l = 0.80 m (the length of the rod); v = 5.0 m/s; B = 0.040 T. Question: ΔV.

2. ΔV = Blv = (0.040)(0.80)(5.0) = 0.16 V.

3. Answer: 0.160 V

Medium · B15-N04. A 0.50 m rod moves at 12 m/s at right angles to a 0.25 T field. What potential difference appears between its ends?

1. Givens: l = 0.50 m (the length of the rod); v = 12 m/s; B = 0.25 T. Question: ΔV.

2. ΔV = Blv = (0.25)(0.50)(12) = 1.5 V.

3. Answer: 1.50 V

B15 · Worked practice

The generator effect and Lenz’s law | Questions 5-6 of 8

Medium · B15-N05. A magnet is pushed into a coil and the meter shows a current. List three changes that would make the current larger.

1. Givens: magnet and coil with a meter. Question: three ways to increase the current.

2. Push the magnet in faster.

3. Use a stronger magnet.

4. Use a coil with more turns (also: a coil with a smaller resistance).

Medium · B15-N06. Why is a generator harder to turn when it is connected to a lamp?

A. The induced current gives a magnetic force that opposes the motion.
B. The lamp makes the magnet heavier.
C. The lamp removes the magnetic field.
D. The coil stops being a conductor.

1. Givens: generator, with and without a lamp. Question: why harder to turn.

2. With the lamp, there is a closed circuit and a current. The current in the field feels a force that opposes the motion (Lenz’s law).

3. The extra work you do becomes the electrical energy in the lamp.

4. Answer: A. The induced current gives a magnetic force that opposes the motion.

B15 · Worked practice

Challenging · B15-N07. A magnet is dropped from rest through 0.30 m in air. Find the free-fall time. Then compare it with 2–3 s for the same magnet inside a vertical aluminium tube. Use g = 9.81 m/s². Give the free-fall time in seconds.

1. Givens: $\Delta d = 0.30$ m; $v_0 = 0$; $a = 9.81$ m/s². Question: t in free fall; comparison with the tube.

2. $t = \sqrt{(2\Delta d/a)} = \sqrt{(2(0.30) \div 9.81)} = 0.25$ s.

3. In the tube the fall takes about ten times longer. The moving magnet induces currents in the aluminium and their magnetic force opposes the fall (Lenz’s law).

4. Answer: 0.247 s

Challenging · B15-N08. A magnet is dropped through a plastic tube, an aluminium tube and a copper tube of the same size. Rank the fall times from shortest to longest and explain.

1. Givens: three tubes, same size. Question: rank the times and explain.

2. Plastic < aluminium < copper.

3. Plastic is an insulator, so no current is induced and the magnet falls freely. Aluminium and copper are conductors, so currents are induced and the force opposes the fall. Copper is the better conductor (lower resistance), so its current and the opposing force are larger, so the fall is slowest.