B12 · Extra practice, worked solutions

Charged particles on circular paths | Diploma-style set of 8

Easy · B12-X01. A proton moves at 2.00×10^{5} m/s at right angles to a field of 0.500 T. Use $e = 1.60 \times 10^{-19}$ C and the proton's mass $m_p = 1.67 \times 10^{-27}$ kg. Find the radius of its circular path, in metres.

1. Givens: $v = 2.00 \times 10^{5}$ m/s; $B = 0.500$ T; $q = 1.60 \times 10^{-19}$ C; $m = 1.67 \times 10^{-27}$ kg. Question: r.

2. The magnetic force is the net force toward the centre: $qvB = mv^2/r$, so $r = mv \div (qB)$.

3. $r = (1.67 \times 10^{-27})(2.00 \times 10^{5}) \div ((1.60 \times 10^{-19})(0.500)) = 4.18 \times 10^{-3}$ m.

4. Answer: 4.18×10^{-3} m

Easy · B12-X02. A charged particle moves in a circle in a magnetic field. The field strength is doubled and nothing else changes. What happens to the radius?

A. It is cut in half.
B. It doubles.
C. It stays the same.
D. It becomes four times larger.

1. Givens: $r = mv/(qB)$; B doubles. Question: the new radius.

2. B is in the denominator, so doubling B halves r.

3. Answer: A. It is cut in half.

Medium · B12-X03. An electron moves at 1.50×10^{7} m/s at right angles to a field of 2.00×10^{-3} T. Use $e = 1.60 \times 10^{-19}$ C and the electron's mass $m_e = 9.11 \times 10^{-31}$ kg. Find the radius of its path, in metres.

1. Givens: $v = 1.50 \times 10^{7}$ m/s; $B = 2.00 \times 10^{-3}$ T; $q = 1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg. Question: r.

2. $r = mv \div (qB)$.

3. $r = (9.11 \times 10^{-31})(1.50 \times 10^{7}) \div ((1.60 \times 10^{-19})(2.00 \times 10^{-3})) = 0.0427$ m.

4. Answer: 0.0427 m

Medium · B12-X04. A proton starts from rest, is accelerated through 1500 V, then enters a field of 0.250 T at right angles. Use $e = 1.60 \times 10^{-19}$ C and the proton's mass $m_p = 1.67 \times 10^{-27}$ kg. Find the radius of its path, in metres.

1. Givens: starts from rest; $\Delta V = 1500$ V; $B = 0.250$ T; $q = 1.60 \times 10^{-19}$ C; $m = 1.67 \times 10^{-27}$ kg. Question: r.

2. Speed from energy: $v = \sqrt{(2q\Delta V \div m)} = \sqrt{(2 \times (1.60 \times 10^{-19})(1500) \div (1.67 \times 10^{-27}))} = 5.361 \times 10^{5}$ m/s.

3. $r = mv \div (qB) = (1.67 \times 10^{-27})(5.361 \times 10^{5}) \div ((1.60 \times 10^{-19})(0.250)) = 0.0224$ m.

4. Answer: 0.0224 m

Medium · B12-X05. A proton and an alpha particle enter the same magnetic field at right angles with the same speed. The alpha particle has about four times the proton's mass and twice its charge. Which statement is correct?

A. The alpha particle's radius is about half the proton's radius.

B. The alpha particle's radius is about twice the proton's radius.

C. The radii are the same.

D. The alpha particle's radius is about four times the proton's radius.

1. Givens: same v and B; m(alpha) ≈ 4 m(proton); q(alpha) = 2 q(proton). Question: compare the radii.

2. $r = mv \div (qB)$, so r is proportional to $m \div q$ when v and B are the same.

3. The alpha particle has $m \div q = 4 \div 2 = 2$ times the proton's value, so its radius is about twice as large.

4. Answer: B. The alpha particle's radius is about twice the proton's radius.

Challenging · B12-X06. A singly charged ion ($q = 1.60 \times 10^{-19}$ C) moves at 3.00×10^{5} m/s at right angles to a field of 0.400 T. Its path has a radius of 0.0150 m. Find the mass of the ion in kilograms.

1. Givens: $q = 1.60 \times 10^{-19}$ C; $v = 3.00 \times 10^{5}$ m/s; $B = 0.400$ T; $r = 0.0150$ m. Question: m.

2. $r = mv \div (qB)$, so $m = qBr \div v$.

3. $m = (1.60 \times 10^{-19})(0.400)(0.0150) \div (3.00 \times 10^{5}) = 3.20 \times 10^{-27}$ kg.

4. Answer: 3.20×10^{-27} kg

Challenging · B12-X07. An electron is accelerated from rest through a potential difference, then moves on a circle of radius 0.0200 m in a field of 1.00×10^{-3} T. Use $e = 1.60 \times 10^{-19}$ C and the electron's mass $m_e = 9.11 \times 10^{-31}$ kg. Find the potential difference, in volts.

1. Givens: starts from rest; r = 0.0200 m; $B = 1.00 \times 10^{-3}$ T; $q = 1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg. Question: ΔV.

2. Speed from the radius: $v = qBr \div m = (1.60 \times 10^{-19})(1.00 \times 10^{-3})(0.0200) \div (9.11 \times 10^{-31}) = 3.513 \times 10^{6}$ m/s.

3. Energy: $q\Delta V = \frac{1}{2}mv^2$, so $\Delta V = mv^2 \div (2q) = (9.11 \times 10^{-31})(3.513 \times 10^{6})^2 \div (2 \times 1.60 \times 10^{-19}) = 35.1$ V.

4. Answer: 35.1 V

Challenging · B12-X08. Two ions have the same charge but different masses. They are accelerated through the same potential difference and then enter the same magnetic field. Explain how the field separates them.

1. Givens: two ions with the same charge, different mass; same ΔV; same B. Question: how the field separates them.

2. Both gain the same kinetic energy qΔV, but the heavier ion has the smaller speed: $v = \sqrt{(2q\Delta V/m)}$.

3. The radius is $r = mv/(qB)$. Putting in v gives $r = (1/B)\sqrt{(2m\Delta V/q)}$, so a larger mass gives a larger radius.

4. The two ions follow circles of different radius and land in different places. Measuring the radius shows the mass.