B12 · Charged particles on circular paths

Completed notes

Find the radius of a circular path, r = mv/(qB), including after acceleration through a voltage.

Slide 1 of 5 · Why the path is a circle

The magnetic force is at **right angles** to the velocity. No part of the force points along the motion, so it cannot change the **speed**. It only changes the **direction**.

The force always points to one **centre**, so the path is a **circle**. This is uniform **circular** motion. Outside the field the path is **straight**.

The net force toward the centre is the **centripetal** force. Here it is the **magnetic** force. On the diploma: the **speed** stays the same in a magnetic field.

To find which way a particle curves, use the right-hand rule at the first moment (reverse it for an **electron**). The path bends toward the **force**.

Example. An electron moves up the page. A uniform magnetic field covers the whole page and points out of the page, toward you. Does the electron start to curve toward the left side or the right side of the page?

1. For a positive charge moving up through a field out of the page, the right hand gives a force to the right.

2. The electron is negative, so the force is reversed: to the left.

3. The path bends toward the left side of the page.

Try it. An electron enters a region with an electric field between charged plates, and an identical electron enters a magnetic field at the same speed, both moving at right angles to the field. Compare the shape of the two paths, and say what happens to the speed in each.

1. Between the plates the force on the electron always points the same way, so the path is a parabola and the speed increases.

2. In the magnetic field the force is always at right angles to the velocity, so the path is a circle and the speed stays constant.

Slide 2 of 5 · The radius of the circle: r = mv/(qB)

The magnetic force is the net force on the circle. Newton's second law: qvB = **mv²/r**. Divide both sides by v: qB = **mv/r**. Solve for r: r = **mv/(qB)**.

B is the field strength, in **tesla** (T). Put in the **size** of q. The velocity is at **right angles** to B. This formula is **not** on the data sheet, so show where it comes from.

A bigger v or m makes a **bigger** circle. A bigger q or B makes a **smaller** circle.

Use SI units: **kilograms**, metres per second, **coulombs** and tesla. On the diploma: explain a change in the radius using **ratios**.

Example. A proton (mass 1.67×10^{-27} kg, charge 1.60×10^{-19} C) moves at 3.2×10^{5} m/s at right angles to a magnetic field of 0.150 T. What is the radius of its circular path?

1. $qvB = mv^2/r$, so $r = mv/(qB)$.

2. $r = (1.67 \times 10^{-27}\ \text{kg})(3.2 \times 10^{5}\ \text{m/s})/[(1.60 \times 10^{-19}\ \text{C})(0.150\ \text{T})]$

3. $r = 2.23 \times 10^{-2}$ m.

Try it. An electron (mass 9.11×10^{-31} kg, charge size 1.60×10^{-19} C) moves to the right at 6.0×10^{6} m/s at right angles to a field of 2.0×10^{-3} T that points out of the page. What is the radius of its path, and does it circle clockwise or counterclockwise on the page?

1. $r = mv/(qB) = (9.11 \times 10^{-31}\ \text{kg})(6.0 \times 10^{6}\ \text{m/s})/[(1.60 \times 10^{-19}\ \text{C})(2.0 \times 10^{-3}\ \text{T})] = 1.71 \times 10^{-2}$ m.

2. The field is out of the page and the electron moves right. The right hand gives a force down for a positive charge, so the force on the electron is up.

3. The force points toward the centre, so the centre is above the electron and the path turns toward the left: counterclockwise.

Slide 3 of 5 · Accelerate first, then bend

Two stages: a voltage speeds the particle up, then a field bends it. Stage one: the charge gains energy **qΔV** (B6), so $q\Delta V = \mathbf{\frac{1}{2}mv^2}$. This gives the **speed**. ΔV is the voltage, in volts. The particle starts from **rest**.

Stage two: $r = \mathbf{mv/(qB)}$, using the speed from stage one. Use the **size** of q.

Keep **extra** digits in the speed. Use SI units and the **same** charge in both stages. On the diploma: **label** your two stages.

Example. An electron starts at rest and is accelerated through 150 V, then enters a magnetic field of 1.2×10^{-3} T at right angles to its velocity. The electron's mass is 9.11×10^{-31} kg and its charge size is 1.60×10^{-19} C. What is the radius of its path in the magnetic field?

1. Stage one: $q\Delta V = \frac{1}{2}mv^2$, so $v = \sqrt{(2q\Delta V/m)} = \sqrt{[2(1.60 \times 10^{-19})(150)/(9.11 \times 10^{-31})]} = 7.259 \times 10^{6}$ m/s.

2. Stage two: $r = mv/(qB) = (9.11 \times 10^{-31})(7.259 \times 10^{6})/[(1.60 \times 10^{-19})(1.2 \times 10^{-3})]$

3. $r = 3.44 \times 10^{-2}$ m.

Try it. An alpha particle (mass 6.65×10^{-27} kg, charge 3.20×10^{-19} C) starts at rest and is accelerated through 800 V. It then enters a field of 0.0500 T at right angles. What is the radius of its path in the field?

1. Stage one: $v = \sqrt{(2q\Delta V/m)} = \sqrt{[2(3.20 \times 10^{-19})(800)/(6.65 \times 10^{-27})]} = 2.775 \times 10^{5}$ m/s.

2. Stage two: $r = mv/(qB) = (6.65 \times 10^{-27})(2.775 \times 10^{5})/[(3.20 \times 10^{-19})(0.0500)]$

3. $r = 0.115$ m. Use the full alpha charge, 3.20×10^{-19} C, in both stages.

Slide 4 of 5 · Method: how to solve a circular-path problem

Step 1: **split** the problem. Is there an accelerator first? Mark the plates (energy) and the field (**circle**), and name the unknown.

Step 2: find the speed from the energy: $qΔV = \mathbf{½mv^2}$, so $v = \mathbf{\sqrt{(2qΔV/m)}}$, from rest. If the speed is given, **skip** this step. Keep the full value.

Step 3: set the magnetic force equal to the **centripetal** force: $qvB = \mathbf{mv^2/r}$. Solve for the unknown (r, v, m or q) and use the **size** of q.

Step 4: calculate with units: kilograms, **metres per second**, coulombs and tesla. Put in the **full** speed from step 2.

Step 5: find which way it turns. Use the right-hand rule for the first force (reverse it for a **negative** charge). The **centre** is on that side. Check: a faster or heavier particle makes a **bigger** circle, and more charge or field makes a **smaller** one.

Example. An electron starts at rest and is accelerated through 150 V. It then moves to the right into a field of 1.2×10^{-3} T, into the page. Find the radius of its path and which way it turns. ($m = 9.11 \times 10^{-31}$ kg, $q = 1.60 \times 10^{-19}$ C)

1. Step 1: the plates accelerate the electron and the field bends it. The unknowns are r and the turn.

2. Step 2: $v = \sqrt{(2qΔV/m)} = \sqrt{[2(1.60 \times 10^{-19})(150)/(9.11 \times 10^{-31})]} = 7.259\ldots \times 10^{6}$ m/s.

3. Step 3: $qvB = mv^2/r$, so $r = mv/(qB)$. Step 4: $r = (9.11 \times 10^{-31})(7.259 \times 10^{6})/[(1.60 \times 10^{-19})(1.2 \times 10^{-3})] = 3.444 \times 10^{-2}$ m, so $r = 3.44 \times 10^{-2}$ m.

4. Step 5: a positive charge moving right through a field into the page is pushed up. The electron is pushed down, so the centre is below it and the path turns clockwise.

Try it. NEW, for teacher review. A proton starts at rest and is accelerated through 2.00×10^{3} V. It then moves to the right into a field of 0.0400 T, out of the page. Find the radius of its path and which way it turns. ($m = 1.67 \times 10^{-27}$ kg, $q = 1.60 \times 10^{-19}$ C)

1. Step 2: $v = \sqrt{(2qΔV/m)} = \sqrt{[2(1.60 \times 10^{-19})(2.00 \times 10^{3})/(1.67 \times 10^{-27})]} = 6.1906\ldots \times 10^{5}$ m/s.

2. Step 3: $r = mv/(qB)$. Step 4: $r = (1.67 \times 10^{-27})(6.1906 \times 10^{5})/[(1.60 \times 10^{-19})(0.0400)] = 0.1615$ m, so $r = 0.162$ m.

3. Step 5: a positive charge moving right through a field out of the page is pushed down, so the centre is below it and the path turns clockwise.

4. Check: the proton has much more mass than the electron, so its circle is bigger.

Slide 5 of 5 · Using the radius to find the charge

Run the formula backward. Start from qvB = **mv²/r** and solve for the unknown. For the charge, q = **mv/(rB)**. For the field, B = **mv/(qr)**.

Ask what multiplies the unknown. Do the **opposite** to both sides. Check: put the answer back into r = mv/(qB). Is the **radius** the same?

Charge = n × e (B1). Here e = **1.60 × 10⁻¹⁹ C** and n is a whole number of electrons. So n = q ÷ **e**. A negative charge means n **extra** electrons.

The calculator will not give an exact whole number, so write n as a **whole** number. On the diploma: write each rearranging step.

Example. A negatively charged particle of mass 9.6×10^{-27} kg moves at 3.0×10^{5} m/s on a circle of radius 0.0144 m in a field of 0.250 T. What is its charge, and how many extra electrons does it have? ($e = 1.60 \times 10^{-19}$ C)

1. $qvB = mv^2/r$, so $q = mv/(rB)$.

2. $q = (9.6 \times 10^{-27})(3.0 \times 10^{5})/[(0.0144)(0.250)] = 8.00 \times 10^{-19}$ C.

3. Number of electrons = $8.00 \times 10^{-19} \div (1.60 \times 10^{-19}) = 5.00$, so the particle has 5 extra electrons.

Try it. A proton (mass 1.67×10^{-27} kg, charge 1.60×10^{-19} C) moves at 2.4×10^{5} m/s on a circle of radius 0.025 m. What is the strength of the magnetic field?

1. $qvB = mv^2/r$ gives $B = mv/(qr)$.

2. $B = (1.67 \times 10^{-27}\ \text{kg})(2.4 \times 10^{5}\ \text{m/s})/[(1.60 \times 10^{-19}\ \text{C})(0.025\ \text{m})]$

3. B = 0.100 T. Check: r = mv/(qB) gives back 0.025 m.

B12 · Worked practice

Charged particles on circular paths | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}\ N \cdot m^2/C^2$, $e = 1.60 \times 10^{-19}\ C$, electron mass $m_e = 9.11 \times 10^{-31}\ kg$, proton mass $mp = 1.67 \times 10^{-27}\ kg$, $g = 9.81\ m/s^2$.

Easy · B12-N01. Why does a charged particle move in a circle when it enters a magnetic field at right angles to the field?

1. Givens: velocity □ B. Question: why a circle.

2. The magnetic force is perpendicular to the velocity, so it changes only the direction of the velocity.

3. The force has constant size and is always toward the centre, so it is a centripetal force and the path is a circle.

Easy · B12-N02. An electron moves at 2.0×10^{6} m/s at right angles to a field of 0.0400 T. Find the radius of its path. Use $e = 1.60 \times 10^{-19}$ C and the electron's mass $m_e = 9.11 \times 10^{-31}$ kg.

1. Givens: $m = 9.11 \times 10^{-31}$ kg; $v = 2.0 \times 10^{6}$ m/s; $q = 1.60 \times 10^{-19}$ C; $B = 0.0400$ T. Question: r.

2. $r = mv/(qB) = (9.11 \times 10^{-31})(2.0 \times 10^{6}) \div ((1.60 \times 10^{-19})(0.0400)) = 2.85 \times 10^{-4}$ m.

3. Answer: 2.85×10^{-4} m

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Easy · B12-N03. A proton moves in a circle in a magnetic field. The field strength is doubled, with the same speed. What happens to the radius?

A. It doubles.
B. It is unchanged.
C. It becomes four times smaller.
D. It is halved.

1. Givens: B doubled; v, m, q unchanged. Question: change in r.

2. r = mv/(qB) so r depends on 1/B. Doubling B halves r.

3. Answer: D. It is halved.

Medium · B12-N04. A proton moving at 1.5×10^{5} m/s enters a field of 0.0800 T at right angles. Find the radius of its circular path. Use the proton's mass mp = 1.67×10^{-27} kg.

1. Givens: m = 1.67×10^{-27} kg; v = 1.5×10^{5} m/s; q = 1.60×10^{-19} C; B = 0.0800 T. Question: r.

2. r = mv/(qB) = $(1.67 \times 10^{-27})(1.5 \times 10^{5}) \div ((1.60 \times 10^{-19})(0.0800))$ = 0.0196 m.

3. Answer: 0.0196 m

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Medium · B12-N05. A proton starts at rest and is accelerated through 250 V. It then enters a field of 0.250 T at right angles. Find the radius of its path.

1. Givens: $v_0 = 0$; $\Delta V = 250$ V; $B = 0.250$ T; $q = 1.60 \times 10^{-19}$ C; $m = 1.67 \times 10^{-27}$ kg. Question: r.

2. Step 1, speed: $v = \sqrt{(2q\Delta V/m)} = \sqrt{[2(1.60 \times 10^{-19})(250) \div (1.67 \times 10^{-27})]} = 2.189 \times 10^5$ m/s. Keep the full value on your calculator.

3. Step 2, radius: $r = mv/(qB) = (1.67 \times 10^{-27})(2.189 \times 10^5) \div ((1.60 \times 10^{-19})(0.250)) = 9.14 \times 10^{-3}$ m.

4. Answer: 9.14×10^{-3} m

Medium · B12-N06. An electron and a proton enter the same field at right angles with the same speed. Which has the larger radius?

A. The proton, because it has the larger mass

B. The electron, because it is lighter

C. They have the same radius because the charges are equal in size

D. The electron, because it is negative

1. Givens: same v, B and |q|; different masses. Question: which has the larger r.

2. $r = mv/(qB)$ so r increases with m. The proton is about 1800 times heavier, so its radius is about 1800 times larger.

3. Answer: A. The proton, because it has the larger mass

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Charged particles on circular paths | Questions 7-8 of 8

Challenging · B12-N07. A particle of mass 6.68×10^{-27} kg moves at 2.0×10^{5} m/s at right angles to a field of 0.500 T. Its path has radius 8.35×10^{-3} m. Find the size of its charge.

1. Givens: $m = 6.68 \times 10^{-27}$ kg; $v = 2.0 \times 10^{5}$ m/s; $B = 0.500$ T; $r = 8.35 \times 10^{-3}$ m. Question: q.

2. Rearrange $r = mv/(qB)$ to $q = mv/(rB)$.

3. $q = (6.68 \times 10^{-27})(2.0 \times 10^{5}) \div ((8.35 \times 10^{-3})(0.500)) = 3.20 \times 10^{-19}$ C.

4. That is 2.0 times e, so the charge is 2e.

5. Answer: 3.20×10^{-19} C

Challenging · B12-N08. Compare the path of a charged particle between two parallel charged plates with its path in a magnetic field at right angles to its velocity. Mention the force, the path and the speed.

1. Givens: two situations. Question: compare force, path and speed.

2. Plates: the force $F = qE$ is constant and along the field, so the path is a parabola (if it enters sideways) and the speed changes.

3. Magnetic field: the force is perpendicular to v and constant in size, so the path is a circle and the speed is constant.