B11 · Magnetic force on a moving charge

Completed notes

Find the size and direction of the magnetic force on a moving charge.

Slide 1 of 5 · What a magnetic force needs

A moving charge carries its own small field. In an outside field the two fields interact and the charge feels a **force**. The force needs a **moving** charge and a **magnetic field**.

Its size is F = **|q|v⊥B**. Here |q| is the **size** of the charge in coulombs, used without its minus sign; v is in **metres per second**; B is in **tesla (T)**, where 1 T = 1 N ÷ (C·m/s). The symbol v⊥ means only the part of the velocity at **right angles** to the field counts.

A charge moving across the field feels the **most** force. A charge moving at an angle feels **less** force. A charge moving along the field lines feels **no** force, and so does a charge that is not moving.

The formula gives only the **size** of the force, so use |q| and decide the direction separately. A **uniform** field has the same size and direction at every point. On the diploma: the data sheet gives the electron's charge, and a numerical response needs **three** significant digits.

Example. An electron moves at 2.5×10^{6} m/s at right angles to a uniform magnetic field of 0.350 T (uniform: the same size and direction at every point). The charge of an electron has size 1.60×10^{-19} C. What is the size of the magnetic force on the electron, in newtons?

1. The electron moves at right angles to the field, so $v_\perp = v = 2.5 \times 10^{6}$ m/s.

2. $F = |q|v_\perp B = (1.60 \times 10^{-19}\ \text{C})(2.5 \times 10^{6}\ \text{m/s})(0.350\ \text{T})$

3. $F = 1.40 \times 10^{-13}$ N.

Try it. A proton (charge 1.60×10^{-19} C) moves at 4.0×10^{6} m/s at right angles to a uniform magnetic field of 0.50 T. What is the size of the magnetic force on it? What would the force be if the proton were at rest?

1. The proton moves at right angles to the field, so $v_\perp = v = 4.0 \times 10^{6}$ m/s.

2. $F = |q|v_\perp B = (1.60 \times 10^{-19}\ \text{C})(4.0 \times 10^{6}\ \text{m/s})(0.50\ \text{T}) = 3.2 \times 10^{-13}$ N.

3. If the proton is at rest, v = 0, so the force is zero. A magnetic force needs a moving charge.

Slide 2 of 5 · Direction: the right-hand rule

Use a flat right hand for this rule (it is a different rule from the curled grip in B10). The magnetic force is at **right angles** to both the velocity and the field. It never points along the **field** or along the **velocity**.

For a positive charge: point your **fingers** along the magnetic field, point your **thumb** along the velocity, and the force pushes out of your **palm**.

Example: the fingers point into the page and the thumb points right, so the palm faces **up** the page and the force on the positive charge is **up** the page.

For a negative charge, find the direction for a **positive** charge first and then **reverse** it. On the diploma: write the positive case first so the reversal is a visible step; a common wrong choice forgets to reverse for an **electron**.

Example. An electron moves to the right through a magnetic field that points out of the page. In which direction is the magnetic force on the electron: up the page, down the page, into the page or out of the page?

1. For a positive charge moving right in a field out of the page, the fingers point at you, the thumb points right and the palm faces down the page, so the force would be down.

2. The electron is negative, so the force is reversed: it points up the page.

Try it. A proton moves straight up the page through a field that points to the left, all in the plane of the page. Which way is the magnetic force on the proton? Which way would it be on an electron moving the same way?

1. The force is at right angles to both vectors, so it cannot be in the plane of the page.

2. Fingers point left (field) and thumb points up (velocity); the palm faces out of the page, so the force on the proton is out of the page.

3. An electron is negative, so the force on it is reversed: into the page.

Slide 3 of 5 · When the motion is at an angle

Split the velocity into a part along the field and a part **across** the field. Only the part across the field, **$v_\perp$**, produces a force.

Draw a right triangle with the velocity as the **long side** (the hypotenuse). Sine = opposite side ÷ **long side**; cosine = side next to the angle ÷ **long side**. Opposite means across from the angle. Set your calculator to **degrees**. If θ is measured between the velocity and the **field**, the part across the field is opposite θ, so $v_\perp = $ **$v \sin \theta$**.

If the angle is measured from the direction **across** the field, that part is next to the angle, so use **cosine**: $v_\perp = v \cos \theta$. Draw the picture and decide which side is across the field before choosing sine or cosine.

Then use $F = $ **$|q|v_\perp B$**. Check that $v_\perp$ is never **bigger** than v. On the diploma: bearing problems use a compass, so **sketch** it; sine versus cosine is a favourite wrong choice.

Example. An electron travels along the ground at 1.8×10^6 m/s in the direction 25° east of south (face south, then turn 25° toward east). The magnetic field is 0.150 T and points east. Use $e = 1.60 \times 10^{-19}$ C. What is the size of the magnetic force on the electron?

1. The field is east. The component of the velocity pointing south is at right angles to it, and it is next to the 25° angle, so $v_\perp = v \cos 25° = (1.8 \times 10^6)(0.90631) = 1.6314 \times 10^6$ m/s.

2. $F = |q|v_\perp B = (1.60 \times 10^{-19})(1.6314 \times 10^6)(0.150) = 3.92 \times 10^{-14}$ N.

Try it. A proton enters a uniform field of 0.50 T moving at 3.0×10^6 m/s at 30° to the field direction. Use $q = 1.60 \times 10^{-19}$ C. What is the size of the force, and what is the force if the proton enters exactly along the field?

1. The angle is measured from the field, so $v_\perp = v \sin 30° = (3.0 \times 10^6)(0.50) = 1.5 \times 10^6$ m/s.

2. $F = |q|v_\perp B = (1.60 \times 10^{-19})(1.5 \times 10^6)(0.50) = 1.2 \times 10^{-13}$ N.

3. Along the field the angle is zero, so $v_\perp = 0$ and the force is zero.

Slide 4 of 5 · Method: how to find the force on a moving charge

Step 1, find **$v_\perp$**: draw the field and the velocity. Only the part of v **across** the field counts. $v_\perp = v$ **sin θ** when θ is measured from the field, and v **cos θ** when it is measured from across the field.

Step 2, find the **size**: $F = |q|v_\perp B$. Use the size of the charge, with no minus sign.

Step 3, find the direction: right hand, **fingers** along the field, **thumb** along the velocity, and the palm gives the force on a positive charge. For a negative charge, **reverse** it.

Step 4, look for **other** forces: if a mass is given, gravity **mg** also acts. “Not deflected” means the magnetic force and gravity **cancel**.

Check: $v_\perp$ can never be **bigger** than v, so the force is never more than **$|q|vB$**. The force is zero along the **field**.

Example. A proton (charge $+1.60 \times 10^{-19}$ C) moves at 4.0×10^{6} m/s at 35° to a magnetic field of 0.620 T. What is the size of the magnetic force on it? (The slide ran through this question.)

1. Step 1: the angle is measured from the field, so $v_\perp = v \sin 35° = (4.0 \times 10^{6})(0.57358) = 2.2943... \times 10^{6}$ m/s. Keep the full value on your calculator.

2. Step 2: $F = |q|v_\perp B = (1.60 \times 10^{-19})(2.2943... \times 10^{6})(0.620) = 2.276... \times 10^{-13}$ N, so $F = 2.28 \times 10^{-13}$ N.

3. Steps 3 and 4: only the size is asked, so no direction is needed, and no mass is given, so there is no gravity to add.

Try it. NEW, for teacher review. An electron moves at 2.0×10^{6} m/s at 40° to a uniform magnetic field of 0.30 T. Use $e = 1.60 \times 10^{-19}$ C. What is the size of the magnetic force on the electron? Write the step number beside each line.

1. Step 1: the angle is measured from the field, so $v_\perp = v \sin 40° = (2.0 \times 10^{6})(0.64279) = 1.2856... \times 10^{6}$ m/s.

2. Step 2: $F = |q|v_\perp B = (1.60 \times 10^{-19})(1.2856... \times 10^{6})(0.30) = 6.171... \times 10^{-14}$ N, so $F = 6.17 \times 10^{-14}$ N.

3. Steps 3 and 4: only the size is asked, and no mass is given. Check: 6.17×10^{-14} N is less than the full $|q|vB = 9.60 \times 10^{-14}$ N, as it must be.

Slide 5 of 5 · Magnetic force against gravity

If a question gives the mass of a particle and asks about forces, **gravity** is part of the story. Gravity on the particle is **mg**, down, with g = 9.81 N/kg (the same as **9.81 m/s²**).

If the particle is not deflected, the net force is **zero**. So the magnetic force equals gravity in size and is opposite in direction: |Fm| = **|Fg|**. Bars mean **size only** and an arrow means size and direction: the two forces have equal size but are not the same vector.

Then |q|vB = **mg**, so v = **mg/(|q|B)**. Write the balance equation as the **first** line of your solution.

When there is only one force, F⃗net = F⃗m is fine. On the diploma: “not deflected” or “straight line” means net force **zero**. If you cannot rearrange, test options with **units**: a speed must end up in m/s.

Example. A particle of mass m and charge q moves horizontally through a magnetic field B that is at right angles to its velocity. It moves in a straight line, because the magnetic force and the force of gravity balance. Which expression gives its speed? A. qB/(mg). B. mg/(qB). C. mgqB. D. qBm/g. Choose one letter.

1. Answer B.

2. A straight path means the net force is zero, so the magnetic force equals the force of gravity in size: qvB = mg.

3. Solving gives v = mg/(qB). The other options do not come from rearranging that equation.

Try it. A particle has mass 6.7×10^{-27} kg and charge $+3.20 \times 10^{-19}$ C and moves horizontally through a horizontal field of 6.0×10^{-5} T at right angles to its path. What speed does it need so that it is not deflected? Use g = 9.81 N/kg.

1. The particle is not deflected, so qvB = mg and v = mg/(qB).

2. v = $(6.7 \times 10^{-27}\ \text{kg})(9.81\ \text{N/kg})/[(3.20 \times 10^{-19}\ \text{C})(6.0 \times 10^{-5}\ \text{T})]$

3. v = 3.42×10^{-3} m/s.

B11 · Worked practice

Magnetic force on a moving charge | Questions 1-2 of 8

Constants from the Alberta Physics 30 data sheets: $k = 8.99 \times 10^{9}$ N·m²/C², $e = 1.60 \times 10^{-19}$ C, electron mass $m_e = 9.11 \times 10^{-31}$ kg, proton mass $m_p = 1.67 \times 10^{-27}$ kg, $g = 9.81$ m/s².

Easy · B11-N01. An electron moves at 3.6×10^{4} m/s at right angles to a magnetic field of 2.50 T. Find the size of the magnetic force on it. Use $e = 1.60 \times 10^{-19}$ C.

1. Givens: $q = 1.60 \times 10^{-19}$ C; $v = 3.6 \times 10^{4}$ m/s; $B = 2.50$ T; $\theta = 90°$. Question: F.

2. $F = |q|vB = (1.60 \times 10^{-19})(3.6 \times 10^{4})(2.50) = 1.44 \times 10^{-14}$ N.

3. Answer: 1.44×10^{-14} N

Easy · B11-N02. A proton moves toward the top of the page through a field that points into the page. In which direction is the magnetic force?

1. Givens: proton (positive); v up the page; B into the page. Question: force direction.

2. Right hand: fingers along B (into the page), thumb along v of the positive charge (up the page).

3. The palm faces to the left, so the force on the proton is to the left (toward the left side of the page).

B11 · Worked practice

Magnetic force on a moving charge | Questions 3-4 of 8

Easy · B11-N03. Why is the magnetic force on a charged particle zero when it moves along a magnetic field line?

A. Because the particle has no charge
B. Because the field is zero along its own line
C. Because the force is cancelled by gravity
D. Because sin 0° = 0, so no component of v is at right angles to B

1. Givens: v parallel to B. Question: why F = 0.

2. $F = qvB \sin\theta$ and $\theta = 0°$, so F = 0.

3. Answer: D. Because sin 0° = 0, so no component of v is at right angles to B

Medium · B11-N04. A positive charge of 2.0×10^{-6} C moves at 5.0×10^{5} m/s through a field of 0.40 T. The velocity makes an angle of 40° with the field. Find the size of the magnetic force.

1. Givens: $q = 2.0 \times 10^{-6}$ C; $v = 5.0 \times 10^{5}$ m/s; B = 0.40 T; $\theta = 40°$. Question: F.

2. $F = |q|vB \sin\theta = (2.0 \times 10^{-6})(5.0 \times 10^{5})(0.40)(\sin 40°) = 0.257$ N.

3. Answer: 0.26 N

B11 · Worked practice

Magnetic force on a moving charge | Questions 5-6 of 8

Medium · B11-N05. A dust grain with charge 4.0×10^{-9} C and mass 2.0×10^{-7} kg moves horizontally through a horizontal field of 0.50 T at right angles to its path. It is not deflected. What is its speed? Use g = 9.81 N/kg (the same as 9.81 m/s²).

1. Givens: $q = 4.0 \times 10^{-9}$ C; $m = 2.0 \times 10^{-7}$ kg; B = 0.50 T; θ = 90°; g = 9.81 N/kg (= 9.81 m/s²). Question: v.

2. Not deflected means the magnetic force balances gravity: $|q|vB = mg$.

3. $v = mg/(|q|B) = (2.0 \times 10^{-7})(9.81) \div ((4.0 \times 10^{-9})(0.50)) = 981$ m/s.

4. Answer: 981 m/s

Medium · B11-N06. A magnetic field acts on a charged particle at right angles to its velocity. What happens to the particle’s speed?

A. It stays constant, because the force is perpendicular to the velocity.

B. It increases, because there is a force.

C. It decreases, because the field opposes motion.

D. It increases and then decreases.

1. Givens: F □ v. Question: what happens to speed.

2. A force at right angles to the velocity does no work, so the kinetic energy and speed are unchanged; only the direction changes.

3. Answer: A. It stays constant, because the force is perpendicular to the velocity.

B11 · Worked practice

Magnetic force on a moving charge | Questions 7-8 of 8

Challenging · B11-N07. An electron moves at 2.0×10^{6} m/s at right angles to a field of 0.0400 T. Find the size of its acceleration. Use $e = 1.60 \times 10^{-19}$ C and the electron's mass $m_e = 9.11 \times 10^{-31}$ kg.

1. Givens: $q = 1.60 \times 10^{-19}$ C; $v = 2.0 \times 10^{6}$ m/s; $B = 0.0400$ T; $m = 9.11 \times 10^{-31}$ kg. Question: a.

2. $F = |q|vB = (1.60 \times 10^{-19})(2.0 \times 10^{6})(0.0400) = 1.280 \times 10^{-14}$ N. Keep the full value on your calculator.

3. By Newton's second law, $a = F \div m = 1.280 \times 10^{-14} \div (9.11 \times 10^{-31}) = 1.41 \times 10^{16}$ m/s². The acceleration is at right angles to the velocity.

4. Answer: 1.41×10^{16} m/s²

Challenging · B11-N08. A positive and a negative particle with the same speed and charge size move in the same direction through the same field at right angles. Compare the size and direction of their magnetic forces.

1. Givens: same |q|, v and B; opposite signs. Question: compare the forces.

2. $F = |q|vB$ is the same for both.

3. The directions are opposite, because reversing the sign of the charge reverses the force.