# B4 · Electric fields

Completed notes

Distinguish field from force, map field directions and combine source fields.

## Slide 1 of 6 · Field means force per charge

In B2 and B3, one charge pushed another across empty space. A field explains how.

A charge can push another charge across empty space. It does this with a **field**. A small test charge, **q_test**, feels a force F. Double q_test and F **doubles**.

So divide out the charge: E = **F/q_test**, the force per unit charge, in **N/C**.

E points the way a **positive** test charge is pushed. A negative charge is pushed the **opposite** way.

F = **|q|E**. The size of q gives the size of F. The **sign** gives the direction.

Convert μC to **C**: multiply by **$10^{-6}$**. Keep extra digits and round **once**, at the end.

**Example.** A +5.00 μC test charge feels a force of 0.0150 N south at a point. What are the size and direction of the electric field at the point?

**1.** Change to coulombs: 5.00 μC = $5.00 \times 10^{-6}$ C.

**2.** E = F/q_test = (0.0150 N) ÷ ($5.00 \times 10^{-6}$ C) = $3.00 \times 10^{3}$ N/C.

**3.** The test charge is positive, so the field points the same way as the force: south.

**Try it.** An electron (e = $1.60 \times 10^{-19}$ C) is placed at a point where the electric field is 500 N/C north. What are the size and direction of the force on the electron?

**1.** Size: F = |q|E = ($1.60 \times 10^{-19}$ C)(500 N/C) = $8.00 \times 10^{-17}$ N.

**2.** The electron is negative, so the force is opposite to the field: south.

## Slide 2 of 6 · The field of a point charge

Slide 1 defined the field. Now find it for a point charge.

A point source **Q** makes a field. r is the distance from the **source**. Coulomb's law gives $F = k|Q|q_{test}/r^2$. Divide by $q_{test}$ and it **cancels**.

$E = \mathbf{k|Q|/r^2}$, with $k = \mathbf{8.99 \times 10^9\ N{\cdot}m^2/C^2}$.

The field depends on the **source** and the **distance**, not on the test charge. Double r: the field is **one quarter**.

Positive source: the field points **away**. Negative source: **toward**. Use |Q| in the formula. The sign gives **direction** only.

Convert prefixes first: μ = $\mathbf{10^{-6}}$, n = $\mathbf{10^{-9}}$. The distance must be in **metres**.

**Example.** A source of +4.0 μC is 0.30 m from point P. Use $k = 8.99 \times 10^9\ N{\cdot}m^2/C^2$. What are the size and direction of the electric field at P?

**1.** The source is positive, so the field at P points away from it.

**2.** $E = k|Q|/r^2 = (8.99 \times 10^9\ N{\cdot}m^2/C^2)(4.0 \times 10^{-6}\ C) \div (0.30\ m)^2 = 4.00 \times 10^5\ N/C$.

**Try it.** A source of −2.0 nC (that is, $-2.0 \times 10^{-9}$ C) is 0.050 m from point P. What are the size and direction of the electric field at P?

**1.** The source is negative, so the field at P points toward it.

**2.** $E = (8.99 \times 10^9)(2.0 \times 10^{-9}) \div (0.050)^2 = 7.19 \times 10^3\ N/C$.

## Slide 3 of 6 · From field to force to acceleration

Slides 1 and 2 gave the field. Now use it to find how a particle moves.

Field gives **force**, and force gives **acceleration**. Here q is the particle's **own** charge, not the test charge.

Force: F = **|q|E**. Only the electric force acts, so this is the net force.

Acceleration: a = **F/m**, so a = **|q|E/m**. Do the force on its own line first.

The force points **along** the field for a positive charge and **against** it for a negative charge. A uniform field has the **same size and direction** everywhere.

An electron has mass **$9.11 \times 10^{-31}$ kg**. Its weight is about $10^{13}$ times smaller than this force, so gravity is **ignored**.

**Example.** An electron ($q = -1.60 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) is placed in a uniform electric field of $2.50 \times 10^{3}$ N/C north. Only the electric force acts. What are the size and direction of the electron's acceleration?

**1.** Force: $F = |q|E = (1.60 \times 10^{-19}\ \text{C})(2.50 \times 10^{3}\ \text{N/C}) = 4.00 \times 10^{-16}$ N. The electron is negative, so the force is south.

**2.** Acceleration: $a = F/m = (4.00 \times 10^{-16}\ \text{N}) \div (9.11 \times 10^{-31}\ \text{kg}) = 4.39 \times 10^{14}\ \text{m/s}^2$, south.

**Try it.** A proton ($q = +1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) is placed in a uniform electric field of $2.50 \times 10^{3}$ N/C north. Only the electric force acts. What are the size and direction of the proton's acceleration?

**1.** Force: $F = |q|E = (1.60 \times 10^{-19}\ \text{C})(2.50 \times 10^{3}\ \text{N/C}) = 4.00 \times 10^{-16}$ N. The proton is positive, so the force is north.

**2.** Acceleration: $a = F/m = (4.00 \times 10^{-16}\ \text{N}) \div (1.67 \times 10^{-27}\ \text{kg}) = 2.40 \times 10^{11}\ \text{m/s}^2$, north.

## Slide 4 of 6 · Method: how to go from field to force to acceleration

These are the steps you used on slide 3.

Step 1: **find** the field E. Use the given field, or $E = \mathbf{k|Q|/r^2}$ for a point source, with the size of Q.

Step 2: name the **direction** of E: away from a **positive** source and toward a **negative** source.

Step 3: find the force with $F = \mathbf{|q|E}$. Here q is the charge placed in the field, not the test charge. Use its **size**.

Step 4: name the direction of F: the **same as** E if q is positive and **opposite to** E if q is negative.

Step 5: find the acceleration with $a = \mathbf{F \div m}$, in the direction of **F**.

Check: the answer has units of $\mathbf{m/s^2}$ and its direction matches the **sign** of q.

**Example.** An electron ($q = -1.60 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) is placed in a uniform electric field of $2.50 \times 10^3$ N/C north. Only the electric force acts. What are the size and direction of the electron's acceleration?

**1.** Step 1 and 2: the field is $E = 2.50 \times 10^3$ N/C, north.

**2.** Step 3: $F = |q|E = (1.60 \times 10^{-19}\text{ C})(2.50 \times 10^3\text{ N/C}) = 4.00 \times 10^{-16}$ N.

**3.** Step 4: the electron is negative, so the force is opposite to the field: south.

**4.** Step 5: $a = F \div m = (4.00 \times 10^{-16}\text{ N}) \div (9.11 \times 10^{-31}\text{ kg}) = 4.39 \times 10^{14}$ m/s², south.

**Try it.** A source of +3.0 µC makes the field at a point 0.30 m away. A −2.0 µC charge of mass 0.0040 kg is placed there. What are the size and direction of its acceleration? Only the electric force acts.

**1.** Step 1: $E = k|Q|/r^2 = (8.99 \times 10^9)(3.0 \times 10^{-6}) \div (0.30)^2 = 2.9967 \times 10^5$ N/C.

**2.** Step 2: the source is positive, so E points away from the source.

**3.** Step 3: $F = |q|E = (2.0 \times 10^{-6})(2.9967 \times 10^5) = 0.5993$ N.

**4.** Step 4: the charge is negative, so F points toward the source.

**5.** Step 5: $a = F \div m = 0.5993 \div 0.0040 = 1.50 \times 10^2$ m/s², toward the source.

## Slide 5 of 6 · Adding fields from several sources

So far one source made the field. Now there are two.

Two sources make two fields. Add them as **vectors**. The sum is the **net field**.

Find each field with $E = \mathbf{k|Q|/r^2}$. Mark each direction at the point. Then add. On a line, call one direction **positive**.

Arrows the same way: the sizes **add**. Opposite ways: the sizes **subtract**. At right angles: size = $\mathbf{\sqrt{(E_1^2 + E_2^2)}}$, tan θ = **opposite ÷ adjacent**.

Field lines: 1. draw the charges and mark their **signs**; 2. start on a **positive** charge, end on a **negative** one; 3. put lines **closer together** where the field is strong; 4. add an **arrow** on every line.

Lines never **cross**. Between large parallel plates the lines are straight and **evenly spaced**: a **uniform** field.

**Example.** Two sources, +2.0 μC at x = 0 and −2.0 μC at x = 0.40 m. Point P is midway between them, 0.20 m from each. Draw the field arrows at P, then find the size and direction of the net field at P.

**1.** The +2.0 μC source pushes a positive test charge east. The −2.0 μC source pulls it east.

**2.** Each field: $E = (8.99 \times 10^9)(2.0 \times 10^{-6}) \div (0.20)^2 = 4.50 \times 10^5$ N/C.

**3.** The fields point the same way, so $E = 9.00 \times 10^5$ N/C east.

**Try it.** Two equal positive point charges are placed on a horizontal line, and point P is midway between them. Which statement about the field at P is correct? A. It is zero. B. It is directed toward the left charge. C. It is directed toward the right charge. D. It is twice the field of one charge.

**1.** Each positive source makes a field at P pointing away from itself.

**2.** The two fields are equal in size and opposite in direction, so they cancel and the field at P is zero. Answer A.

## Slide 6 of 6 · Method: how to add fields from several sources

These are the steps you used on slide 5.

Step 1: **draw** the point and every source, with each source's sign and its distance from the point.

Step 2: find the size of each field on its own with $E = \mathbf{k|Q|/r^2}$.

Step 3: draw each arrow starting at the point, **away from** a positive source and **toward** a negative source.

Step 4: **split** each arrow into x and y parts. Choose a positive direction for each axis and give each part a + or − sign.

Step 5: add all the **x** parts to get Ex and all the **y** parts to get Ey.

Step 6: $E = \sqrt{(E_x^2 + E_y^2)}$. Name the direction from a **reference**, such as east.

Check: the net arrow on the drawing points the **same way** as your answer.

**Example.** Two sources, +2.0 µC at x = 0 and −2.0 µC at x = 0.40 m. Point P is midway between them, 0.20 m from each. East is positive. Find the size and direction of the net field at P.

**1.** Step 1: draw P with +2.0 µC on the left and −2.0 µC on the right, each 0.20 m from P.

**2.** Step 2: each field is $E = (8.99 \times 10^9)(2.0 \times 10^{-6}) \div (0.20)^2 = 4.50 \times 10^5$ N/C.

**3.** Step 3: the + source gives east. The − source gives east.

**4.** Step 4: both x parts are positive. Each y part is zero.

**5.** Step 5: $E_x = 4.50 \times 10^5 + 4.50 \times 10^5 = 9.00 \times 10^5$ N/C. $E_y = 0$.

**6.** Step 6: $E = 9.00 \times 10^5$ N/C east.

**Try it.** Two sources, +4.0 µC at x = 0 and +1.0 µC at x = 0.60 m. East is positive. Point P is 0.20 m from the +4.0 µC source, on the line between the sources. Find the size and direction of the net field at P.

**1.** Step 1: P is 0.20 m from the +4.0 µC source and 0.40 m from the +1.0 µC source.

**2.** Step 2: $E_1 = (8.99 \times 10^9)(4.0 \times 10^{-6}) \div (0.20)^2 = 8.99 \times 10^5$ N/C. $E_2 = (8.99 \times 10^9)(1.0 \times 10^{-6}) \div (0.40)^2 = 5.619 \times 10^4$ N/C.

**3.** Step 3: both sources are positive, so each arrow points away from its source. $E_1$ points east. $E_2$ points west.

**4.** Step 4: x parts: $+8.99 \times 10^5$ and $-5.619 \times 10^4$. Each y part is zero.

**5.** Step 5: $E_x = 8.99 \times 10^5 - 0.5619 \times 10^5 = 8.43 \times 10^5$ N/C.

**6.** Step 6: $E = 8.43 \times 10^5$ N/C east.

# B4 · Worked practice

Electric fields | Questions 1-2 of 12

Use $k = 8.99 \times 10^9\ N \cdot m^2/C^2$ and $g = 9.81\ m/s^2$ where needed. These values are from the Alberta Physics 30 data sheets, Constants section. All point-charge calculations approximate air as vacuum.

Easy · B4-P01. A fixed source creates a field at P. A small positive test charge q_test is replaced by a test charge +2q_test and the source does not change. What happens to the field at P and to the force on the test charge? Explain using the definition of field.

**1.** The source arrangement and point have not changed, so E stays the same.

**2.** F = |q|E gives Fnew = 2|q|E = 2Fold. The force doubles, but F/q_test stays constant. Test charge changes the force, not the field of the source.

Easy · B4-P02. A point source is positive. At a point to its left, draw the electric-field arrow. Then draw the force arrow on a negative test charge placed there. Explain both directions.

**1.** The field points away from a positive source. At the point left of the source, draw E left.

**2.** A negative test charge feels force opposite E, so draw its force right, toward the positive source. The field direction always uses a positive test charge.

# B4 · Worked practice

Electric fields | Questions 3-4 of 12

Easy · B4-P03. A +4.00 µC test charge feels 0.0800 N east in an electric field. What is the field strength and direction at that point?

**1.** Givens: $q_{test} = +4.00 \times 10^{-6}$ C and F = 0.0800 N east from the prompt. Field is force per unit positive test charge.

**2.** $E = F/q_{test} = 0.0800/(4.00 \times 10^{-6}) = 2.00 \times 10^{4}$ N/C east. The positive test charge's force and field directions agree.

Easy · B4-P04. Between large horizontal plates the electric field points up. Ignore the edges of the plates. Which plate is positive, which is negative, and which way is the electric force on an electron? Does the field strength change between two points inside?

**1.** Field arrows run from positive to negative. For an upward field, the lower plate is positive and the upper plate negative.

**2.** The electron is negative, so its electric force points down, opposite the field. In the uniform region between the plates, field strength stays constant from one point to another.

# B4 · Worked practice

Electric fields | Questions 5-6 of 12

Medium · B4-P05. A point source Q = −4.00 μC is fixed in air. P is 0.200 m to its right. What is the electric field at P? Use k = 8.99 × $10^9$ N·$m^2$/$C^2$.

**1.** Givens: |Q| = 4.00 × $10^{-6}$ C and r = 0.200 m from the prompt. k is given. Use E = k|Q|/$r^2$ because one point source creates the field.

**2.** E = (8.99 × $10^9$)(4.00 × $10^{-6}$)/$(0.200)^2$ = 8.99 × $10^5$ N/C.

**3.** A positive test charge at P would be attracted toward the negative source, which is left. Answer: 8.99 × $10^5$ N/C left. P is a point; no second source charge is needed.

Medium · B4-P06. A charge q = −3.00 nC is placed in a field of 4.00 × $10^4$ N/C west. What is the electric force on it? Does its negative sign reverse the field itself?

**1.** Givens: q = −3.00 × $10^{-9}$ C and E = 4.00 × $10^4$ N/C west. Use F = |q|E because the field at the charge is already known.

**2.** |F| = |q|E = (3.00 × $10^{-9}$)(4.00 × $10^4$) = 1.20 × $10^{-4}$ N. The negative charge feels force opposite west: east.

**3.** The field itself is not reversed by choosing a negative charge. The field of the source still points west.

# B4 · Worked practice

Electric fields | Questions 7-8 of 12

Medium · B4-P07. At P, a +2.00 µC source is 0.300 m to the left and a −1.00 µC source is 0.300 m to the right. What is the net electric field at P? Use $k = 8.99 \times 10^9$ N·m²/C².

**1.** Givens are the source charges and distances. The positive source's field points right, away from it. The negative source's field points right, toward it.

**2.** $E_{left} = (8.99 \times 10^9)(2.00 \times 10^{-6})/0.300^2 = 1.9978 \times 10^5$ N/C. $E_{right} = (8.99 \times 10^9)(1.00 \times 10^{-6})/0.300^2 = 9.9889 \times 10^4$ N/C.

**3.** The two fields point the same way, so add them: $E_{net} = 2.9967 \times 10^5 = 3.00 \times 10^5$ N/C right. Adding +2 and −1 µC before finding field directions would give the wrong field.

Medium · B4-P08. A source is at (0,0). A field map shows E at (0.200 m,0) pointing right with strength $4.00 \times 10^5$ N/C. The source stays fixed. What is its sign? Predict the field magnitude and direction at (0.400 m,0) and at (−0.200 m,0).

**1.** At the right-hand point, a field to the right points away from the source, so the source is positive.

**2.** At (0.400 m,0), distance doubles. E follows $1/r^2$ for a point source: $E = (4.00 \times 10^5)/4 = 1.00 \times 10^5$ N/C right.

**3.** At (−0.200 m,0), distance equals the original radius, so strength is $4.00 \times 10^5$ N/C, now left, away from the positive source. Equal distance gives equal magnitude, not necessarily equal direction.

# B4 · Worked practice

Electric fields | Questions 9-10 of 12

Challenging · B4-P09. Point sources A = +1.00 μC at (−0.300 m,0) and C = −2.00 μC at (0,+0.300 m) create a field at P = (0,0). What are the net field magnitude and direction? Use $k = 8.99 \times 10^9$ N·m²/C².

**1.** Givens are the charges and coordinates. At P, the field from positive A points right; the field toward negative C points up. Each source is 0.300 m from P.

**2.** $E_x = (8.99 \times 10^9)(1.00 \times 10^{-6})/0.300^2 = 9.9889 \times 10^4$ N/C. $E_y = (8.99 \times 10^9)(2.00 \times 10^{-6})/0.300^2 = 1.9978 \times 10^5$ N/C.

**3.** $E = \sqrt{(E_x^2 + E_y^2)} = 2.23 \times 10^5$ N/C; $\theta = \tan^{-1}(E_y/E_x) = 63.4°$ north of east. Check: the net field is bigger than the larger component but is below their arithmetic sum.

Challenging · B4-P10. A point has an electric field of $6.00 \times 10^4$ N/C east. A small unknown charge feels 0.120 N west. What are the charge's magnitude and sign? Explain how the directions determine the sign.

**1.** Givens: |F| = 0.120 N and $E = 6.00 \times 10^4$ N/C. Force and field are opposite, so the charge must be negative.

**2.** $|q| = |F|/E = 0.120/(6.00 \times 10^4) = 2.00 \times 10^{-6}$ C. Thus q = −2.00 μC.

**3.** With east positive, $q = F_x/E_x = (-0.120)/(+6.00 \times 10^4)$, which gives the same negative sign. Units N/(N/C) reduce to C.

# B4 · Worked practice

Electric fields | Questions 11-12 of 12

Challenging · B4-P11. Two equal sources are fixed on a horizontal line. Compare the field at the midpoint for (a) two positive sources and (b) a positive source on the left and an equal negative source on the right. Then describe field-line sketches for both pairs: add arrows and explain whether lines cross.

**1.** For two positives, the left source gives a midpoint field to the right and the right source gives an equal field to the left. Net E = 0 at the midpoint. Draw lines leaving both sources and bending away from the space between them; no line joins one positive source to the other.

**2.** For the opposite pair, both midpoint contributions point right: away from the positive and toward the negative. Net E is not zero and points right. Draw lines running from the positive source to the negative source, including curved paths above and below the pair.

**3.** Neither diagram has crossing lines. Crossing would assign two field directions to one point. At the zero-field point in case (a), the net field has no defined direction; do not draw an arrow through it.

Challenging · B4-P12. Mixed review: two identical metal spheres on insulating stands start with +6.00 µC and −2.00 µC, touch, then separate far apart. Afterwards one sphere is held fixed as a point source; the other is moved far enough away that its field at P is too small to matter. P is 0.300 m right of the fixed sphere. What field is at P, and what force acts on a −2.00 nC charge there? Use $k = 8.99 \times 10^9$ N·m²/C².

**1.** First conserve charge: total = +6.00 - 2.00 = +4.00 µC. Identical spheres share equally, so the fixed source has Q = +2.00 µC. Do not use its initial +6.00 µC charge.

**2.** Use the point-source field: $E = k|Q|/r^2 = (8.99 \times 10^9)(2.00 \times 10^{-6})/0.300^2 = 1.9978 \times 10^5$ N/C, which is $2.00 \times 10^5$ N/C to three digits. The field points right, away from the positive source. Keep the full value on your calculator.

**3.** Use the full field value for the force: $|F| = (2.00 \times 10^{-9})(1.9978 \times 10^5) = 4.00 \times 10^{-4}$ N. The charge is negative, so force is left. The question combines charge conservation, source field and force on a charge.