B3 · Net electric force

Completed notes

Add force vectors on one target and choose between a charge model and force balance.

Slide 1 of 5 · One target, several sources

B2 found the force between two charges. Most real cases have more than two. This lesson adds the forces on one charge.

Pick one charge as the **target**. Every other charge is a **source**. Find the force on the target from **each source**. The sum of the separate forces is the **net force**.

Coulomb's law $F = \mathbf{k|q_1||q_2| \div r^2}$ gives the **size** only, with $k = \mathbf{8.99 \times 10^9\ N{\cdot}m^2/C^2}$. The direction comes from the signs: **alike** charges push apart, **unlike** charges pull together.

Draw an arrow for each force first. Choose east as **positive**. An arrow east counts as **+** and an arrow west counts as **−**. Add the forces **with their signs**. The sign of the net force gives its direction.

The distance r always runs from the **source** to the **target**. If the target is outside the pair of sources, measure a **distance** for each source. Label every position in a sketch before you calculate.

Example. A is +4.0 µC at x = 0, the target B is +2.0 µC at x = 0.30 m, and C is +3.0 µC at x = 0.50 m. East is positive. What is the net force on B, with its direction?

1. A is west of B and alike, so A pushes B east: $F = (8.99 \times 10^9)(4.0 \times 10^{-6})(2.0 \times 10^{-6}) \div (0.30)^2 = +0.799$ N. C is 0.20 m east of B and alike, so C pushes B west: $F = (8.99 \times 10^9)(3.0 \times 10^{-6})(2.0 \times 10^{-6}) \div (0.20)^2 = -1.35$ N.

2. Net = +0.7991 − 1.3485 = −0.5494 N.

3. Net force on B: 0.549 N west (the minus sign means west).

Try it. Same positions, but C is now −3.0 µC. A is +4.0 µC at x = 0, the target B is +2.0 µC at x = 0.30 m, and C is at x = 0.50 m. What is the net force on B, with its direction?

1. A is positive and west of B, so it pushes B away, east: +0.799 N.

2. C is negative and east of B, so it pulls B toward it, also east: +1.35 N.

3. Net force = (+0.799 N) + (+1.35 N) = +2.15 N, so 2.15 N east. Two arrows the same way give a bigger answer.

Slide 2 of 5 · Forces at right angles

Slide 1 had forces on one line. Now two forces meet at a right angle.

A coordinate pair such as (0.30 m, 0) means **0.30 m east and 0 m north**. East is the positive x direction and **north** is the positive y direction.

Find each force with Coulomb's law, and decide each direction from the signs. Draw both arrows **from the target**. If the two forces are at **right angles**, draw a right triangle with the arrows as its sides.

The net force is the **hypotenuse**: $F = \sqrt{(F_1^2 + F_2^2)}$. The angle comes from $\tan\theta =$ **opposite ÷ adjacent**, so $\theta =$ **$\tan^{-1}$(opposite ÷ adjacent)**.

A direction needs an angle and a **reference**, such as "36.9° north of west". If the forces are at 90°, the net force is bigger than **either force** but smaller than their **sum**.

Example. The target (+2.0 µC) is at the origin (0, 0). A +3.0 µC charge is at (0.30 m, 0) and a −4.0 µC charge is at (0, 0.40 m). What is the net force on the target, with its direction?

1. The +3.0 µC charge is alike, so it pushes the target west: $F = (8.99 \times 10^9)(3.0 \times 10^{-6})(2.0 \times 10^{-6}) \div (0.30)^2 = 0.599$ N. The −4.0 µC charge is unlike, so it pulls the target north: $F = (8.99 \times 10^9)(4.0 \times 10^{-6})(2.0 \times 10^{-6}) \div (0.40)^2 = 0.450$ N.

2. The forces meet at a right angle: $F = \sqrt{(0.599^2 + 0.450^2)} = 0.749$ N.

3. $\tan\theta = 0.450 \div 0.599$, so $\theta = 36.9°$. Net force: 0.749 N at 36.9° north of west.

Try it. Same positions, but the charge at (0.30 m, 0) is −3.0 µC and the charge at (0, 0.40 m) is +4.0 µC. The target is still +2.0 µC. What is the net force on the target, with its direction?

1. The sizes do not change: 0.599 N and 0.450 N. Only the directions change.

2. The −3.0 µC charge is east of the target and unlike, so it attracts the target east. The +4.0 µC charge is north of the target and alike, so it repels the target south.

3. Net force = $\sqrt{(0.599^2 + 0.450^2)} = 0.749$ N at $\tan^{-1}(0.450 \div 0.599) = 36.9°$ south of east.

Slide 3 of 5 · Method: how to find the net force on one charge

These are the steps you used on slides 1 and 2, in one place.

Step 1, **draw**: sketch the target and every source. Label each charge and each **distance**.

Step 2, **size**: for each source, F = $\mathbf{k|q_1||q_2| \div r^2}$, using the **sizes** of the charges only. Step 3, **arrow**: alike charges push the target **away**; unlike charges pull it **toward** the source.

Step 4, **split**: write each force as an **x part** and a **y part**. East and north are **+**; west and south are **−**. Step 5, **add**: add all the x parts, then add all the **y parts**.

Step 6, **finish**: size = $\mathbf{\sqrt{x^2 + y^2}}$; angle = $\mathbf{\tan^{-1}(y \div x)}$. Name the direction from a **reference**, such as “north of west”.

Example. Use the six steps. The target (+2.0 µC) is at (0, 0). A +3.0 µC charge is at (0.30 m, 0) and a −4.0 µC charge is at (0, 0.40 m).

1. Steps 1 to 3: the +3.0 µC charge pushes west with 0.599 N. The −4.0 µC charge pulls north with 0.450 N.

2. Steps 4 and 5: x part = −0.599 N, y part = +0.450 N. One force on each axis, so the sums are the same.

3. Step 6: F = $\sqrt{(0.599^2 + 0.450^2)} = 0.749$ N. $\tan\theta = 0.450 \div 0.599$, so $\theta = 36.9°$ north of west.

Try it. Use the six steps. The target (+1.0 µC) is at (0, 0). A +2.0 µC charge is at (0.20 m, 0) and a +3.0 µC charge is at (0, 0.30 m). Find the net force, with its direction.

1. Steps 2 and 3: $F = (8.99 \times 10^9)(2.0 \times 10^{-6})(1.0 \times 10^{-6}) \div (0.20)^2 = 0.4495$ N, pushing west. $F = (8.99 \times 10^9)(3.0 \times 10^{-6})(1.0 \times 10^{-6}) \div (0.30)^2 = 0.2997$ N, pushing south.

2. Steps 4 and 5: x = −0.4495 N, y = −0.2997 N.

3. Step 6: $F = \sqrt{(0.4495^2 + 0.2997^2)} = 0.540$ N. $\tan\theta = 0.2997 \div 0.4495$, so $\theta = 33.7°$ south of west.

Slide 4 of 5 · A force along a diagonal

On slide 2 each force lay along an axis. Now a force points along a slant.

A source at (0.30 m, 0.40 m) is 0.30 m east and 0.40 m north of a target at the origin. The straight-line distance is r = **$\sqrt{(0.30^2 + 0.40^2)} = 0.50$ m**. The force acts along this **diagonal line**.

Find the size first with Coulomb's law. Alike charges push the target **away** from the source and unlike charges pull it toward the source. The target is pushed away, so a source to the north-east pushes the target **south-west**.

A **component** is one part of a force along one axis. The force triangle has the **same shape** as the distance triangle, so component = force × (**side** ÷ **slant side**). Each component is **smaller** than the force.

To add several forces, make two sums: one for **east-west** and one for **north-south**. East and north count as **+**; west and south count as **−**. Then use **$\sqrt{(x^2 + y^2)}$** for the size and tan θ for the angle.

Example. The target (+1.0 μC) is at the origin and a +5.0 μC source is at (0.30 m, 0.40 m). What is the force on the target, and what are its east-west and north-south parts, with directions?

1. $F = (8.99 \times 10^9)(5.0 \times 10^{-6})(1.0 \times 10^{-6}) \div (0.50)^2 = 0.180$ N. The charges are alike, so the target is pushed south-west.

2. West part = 0.180 N × 0.30/0.50 = 0.108 N. South part = 0.180 N × 0.40/0.50 = 0.144 N.

3. Check: $\sqrt{(0.108^2 + 0.144^2)} = 0.180$ N.

Try it. The +1.0 μC target is at the origin and the +5.0 μC source is at (0.30 m, 0.40 m). A second source of −2.0 μC is added at (0, 0.20 m). What is the net force on the target, with its direction?

1. The −2.0 μC source is north of the target and unlike, so it attracts the target north: $F = (8.99 \times 10^9)(2.0 \times 10^{-6})(1.0 \times 10^{-6}) \div (0.20)^2 = 0.450$ N.

2. East-west: only the diagonal source, 0.108 N west. North-south: +0.450 N − 0.144 N = 0.306 N north.

3. Net force = $\sqrt{(0.108^2 + 0.306^2)} = 0.324$ N, at $\tan^{-1}(0.306 \div 0.108) = 70.6°$ north of west.

Slide 5 of 5 · A charge hanging at rest

So far every force was electric. Now gravity and a thread pull as well.

Weight W = **mg**, with g = **9.81 m/s²**: each kilogram weighs 9.81 N. T is the **tension**, the pull of the thread. F_e is the **electric force**, which is horizontal.

At rest the net force is **zero**. The three force arrows close into a **right triangle**. Draw the weight straight **down**, the tension along the **thread** and the electric force **sideways**.

In the triangle, θ is the angle between the weight and the **thread**. F_e is **opposite** θ and mg is adjacent to θ: tan θ = **F_e ÷ mg**, so F_e = **mg tan θ**.

The angle θ is measured from the **vertical**. F_e is **opposite** the angle and mg is **adjacent** to it, so use **tan θ**, not sine or cosine. For two equal charges, Coulomb's law then gives q = **$\sqrt{(F_e r^2 \div k)}$**.

Example. A sphere of mass 0.0100 kg hangs from a thread at 20.0° from the vertical, pushed sideways by a fixed charge 0.15 m away. Find the electric force. If the two charges are equal, find each charge.

1. The sphere is at rest, so weight, tension and electric force form a closed right triangle, and tan θ = F_e ÷ mg.

2. F_e = (0.0100 kg)(9.81 m/s²)(tan 20.0°) = 0.0357 N. No charges were needed.

3. Equal charges: $q = \sqrt{[(0.0357)(0.15)^2 \div (8.99 \times 10^9)]} = 3.0 \times 10^{-7}$ C.

Try it. A sphere of mass 0.0200 kg hangs from a thread and is pushed sideways by a fixed charge so that the thread makes 30.0° with the vertical. The sphere is at rest. What is the size of the electric force on the sphere? Use g = 9.81 m/s².

1. The sphere is at rest, so the weight, tension and electric force form a closed right triangle. The angle is between the weight and the thread, so F_e is opposite it and mg is adjacent: tan θ = F_e ÷ mg.

2. F_e = mg tan θ = (0.0200 kg)(9.81 m/s²)(tan 30.0°) = 0.113 N.

B3 · Worked practice

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Use $k = 8.99 \times 10^9$ N·m²/C² and $g = 9.81$ m/s² where needed. These values are from the Alberta Physics 30 data sheets, Constants section. All point-charge calculations approximate air as vacuum.

Easy · B3-P01. A is positive and left of a negative target B. C is negative and right of B. Draw the two force arrows on B. Do the forces add or partly cancel? Explain each arrow.

1. A attracts the negative B, so the force due to A points left, toward A.

2. C repels the negative B, so the force due to C also points left, away from C. Both arrows act on B and point the same way, so they add. Attraction and repulsion do not automatically mean opposite directions.

Easy · B3-P02. The forces on one charge are 0.600 N right and 0.250 N left. Choose right as positive. What are the signed components and the net force?

1. With right positive, the components are +0.600 N and −0.250 N. Use direction signs rather than charge signs when adding force components.

2. $F_{net,x} = +0.600 - 0.250 = +0.350$ N, so the net force is 0.350 N right. It points with the larger force.

B3 · Worked practice

Net electric force | Questions 3-4 of 12

Easy · B3-P03. A target feels 0.400 N east and 0.400 N west. What is its net force? Does that mean neither source exerts a force? Explain.

1. Choose east positive: $F_{net,x} = +0.400 - 0.400 = 0$ N.

2. Each force is still not zero. Their effects on the target's acceleration cancel. Zero net force is a statement about the vector sum, not that the forces are missing.

Easy · B3-P04. A charged sphere is at rest with its string tilted from vertical by a horizontal electric force. Describe the weight, tension and electric-force arrows. Why must the tension not be drawn vertically?

1. Weight points straight down. Electric force is horizontal. Tension points along the tilted string toward its support, with an upward component and a component opposing the electric force.

2. At rest, the horizontal forces must balance. A purely vertical tension could not oppose the horizontal electric force. The string's actual direction determines the tension arrow.

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Medium · B3-P05. Point charges in air are A = +3.00 μC at x = 0, target B = −1.00 μC at x = 0.300 m and C = +1.00 μC at x = 0.600 m. What is the net force on B? Use $k = 8.99 \times 10^9$ N·m²/C².

1. Givens are the charges and coordinates in the prompt; each source is 0.300 m from B. A attracts B left; C attracts B right. Choose right positive.

2. $F_{A\to B} = k|q_A q_B|/r^2 = (8.99 \times 10^9)(3.00 \times 10^{-6})(1.00 \times 10^{-6})/0.300^2 = 0.2997$ N left. $F_{C\to B} = 0.09989$ N right.

3. $F_{net,x} = -0.2997 + 0.09989 = -0.1998$ N. Answer: 0.200 N left. The stronger attraction wins; the net is less than either-force sum.

Medium · B3-P06. The calculated electric forces on a target are 0.500 N east and 0.200 N north. Draw a vector triangle. What are the size of the net force and its angle north of east?

1. Givens: $F_x = +0.500$ N and $F_y = +0.200$ N. Draw the east vector, then the north vector from its tip; the net force joins the start to the final tip.

2. The components are perpendicular, so $F = \sqrt{(0.500^2 + 0.200^2)} = 0.539$ N. $\theta = \tan^{-1}(0.200/0.500) = 21.8°$ north of east.

3. The result points northeast and is slightly larger than 0.500 N but smaller than 0.700 N. That is consistent with the triangle.

B3 · Worked practice

Net electric force | Questions 7-8 of 12

Medium · B3-P07. A 4.00×10^{-4} kg charged sphere hangs at rest. Its string is 25.0° from vertical and the electric force acts horizontally right. What are the electric-force magnitude and the tension? Use g = 9.81 m/s².

1. Givens: $m = 4.00 \times 10^{-4}$ kg and $\theta = 25.0°$ from the prompt; at rest means a = 0. g is the given data-booklet value. Draw mg down, F_e right and T up-left.

2. Vertical balance: $T \cos \theta = mg$. $T = (4.00 \times 10^{-4})(9.81)/\cos 25.0° = 4.33 \times 10^{-3}$ N.

3. Horizontal balance: $F_e = T \sin \theta = mg \tan \theta = 1.83 \times 10^{-3}$ N right. $F_e < mg < T$, as expected for an angle below 45° from vertical.

Medium · B3-P08. Case A gives two point charges and their distance. Case B gives a hanging sphere's mass and string angle from vertical, with a horizontal electric force. Choose a method to find electric force in each case. Explain why the other method cannot be used directly from the givens.

1. Case A: use Coulomb's law because the charge product and distance determine the force magnitude. Use signs and positions to get direction. No mass or tension data are required.

2. Case B: use force balance. Weight is mg; with angle θ from vertical, $T \cos \theta = mg$ and $F_e = T \sin \theta$, giving $F_e = mg \tan \theta$.

3. Coulomb's law cannot be substituted directly in B because the source charges and the distance are not given. The balance triangle in A would need extra force-balance information.

B3 · Worked practice

Net electric force | Questions 9-10 of 12

Challenging · B3-P09. Point charges in air are target P = +1.00 μC at (0,0), A = +2.00 μC at (−0.300 m,0), and C = +1.00 μC at (0,+0.300 m). What is the net electric force on P, as a magnitude and direction? Use $k = 8.99 \times 10^9$ N·m²/C².

1. Givens are the charges and coordinates. A repels P right; C repels P down. Both source-target distances are 0.300 m. Calculate each force on P separately.

2. $F_x = +(8.99 \times 10^9)(2.00 \times 10^{-6})(1.00 \times 10^{-6})/0.300^2 = +0.1998$ N. $F_y = -(8.99 \times 10^9)(1.00 \times 10^{-6})^2/0.300^2 = -0.09989$ N.

3. $F = \sqrt{(F_x^2 + F_y^2)} = 0.223$ N. $\theta = \tan^{-1}(0.09989/0.1998) = 26.6°$ south of east. The down-right direction comes from the arrows, not from a calculator angle alone.

Challenging · B3-P10. A charged sphere hangs at rest in a horizontal electric force of 3.00×10^{-3} N. Its mass is 5.00×10^{-4} kg. What is the string's angle from vertical and what is the tension? Use g = 9.81 m/s².

1. Givens: $F_e = 3.00 \times 10^{-3}$ N and $m = 5.00 \times 10^{-4}$ kg from the prompt. At rest, acceleration is zero. Weight = mg = 4.905×10^{-3} N.

2. Force balance gives $\tan\theta = F_e/(mg)$, because the horizontal and vertical tension components balance F_e and mg. $\theta = \tan^{-1}(0.00300/0.004905) = 31.5°$ from vertical.

3. $T = \sqrt{(Fe^2 + (mg)^2)} = 5.75 \times 10^{-3}$ N. It is larger than either component, which fits: the tension balances the two together.

B3 · Worked practice

Net electric force | Questions 11-12 of 12

Challenging · B3-P11. Two sources exert electric forces of 0.600 N east and 0.800 N north on a target. A student reports 1.40 N northeast. Find the correct magnitude and direction. Explain exactly what the student’s addition assumes incorrectly.

1. The forces are perpendicular. Use the right triangle: $F = \sqrt{(0.600^2 + 0.800^2)} = 1.00$ N.

2. $\theta = \tan^{-1}(0.800/0.600) = 53.1°$ north of east. “Northeast” alone could suggest 45°, so give the angle.

3. Adding 0.600 + 0.800 treats the forces as parallel and in the same direction. Here the diagonal is smaller than that arithmetic sum and larger than either component.

Challenging · B3-P12. A = +2.00 µC is at x = 0 and C = +8.00 µC is at x = 0.900 m. A positive target charge is placed somewhere between them. Where is the net electric force zero? Give its distance from A and explain why the target’s charge magnitude is not needed.

1. Let x be distance from A, so distance from C is $0.900 - x$. Between like positive sources, A repels the target right and C repels it left. Zero net force requires equal magnitudes.

2. Set $k(2.00\ \mu C)|q|/x^2 = k(8.00\ \mu C)|q|/(0.900 - x)^2$. k and the same |q| cancel. Thus $(0.900 - x)^2 = 4x^2$.

3. Both distances are positive: $0.900 - x = 2x$, so x = 0.300 m from A (0.600 m from C). The larger source is twice as far away, which makes up for its charge being four times as large. The cancelled target charge explains why its magnitude was unnecessary.