B2 · Coulomb’s law

Completed notes

Calculate a pair force, predict changes, and interpret an inverse-square graph.

Slide 1 of 7 · How the force changes

In B1, like charges repelled and unlike charges attracted. This lesson finds how strong that force is.

Symbols: q_1 and q_2 are the two charges in **coulombs (C)**; r is the distance between them in **metres (m)**; F is the size of the force in **newtons (N)**.

Each charge is its own factor. Double one charge: force × **2**. Double both charges: force × 2 × 2 = × **4**.

Distance: F ∝ **1/r²**. The symbol ∝ means **“is proportional to”**. Double r: force × **1/4**. Triple r: force × **1/9**.

Rule: write each change as a **factor**, then **multiply**. For distance, **flip** the change, then **square** it. A graph of F against $1/r^2$ is a **straight line through the origin**.

Example. Two charges exert a force of 0.800 N on each other. Both charges are doubled in size and the distance between them is tripled. What is the new force?

1. Each charge doubles, so the charges give a factor of 2 × 2 = 4.

2. The distance triples: flip to 1/3, then square it: $(1/3)^2 = 1/9$.

3. New force = 0.800 N × 4 ÷ 9 = 0.356 N. The overall factor 4/9 is less than 1, so the force gets smaller.

Try it. Two charges exert a force of 0.150 N on each other. One charge becomes three times larger and the distance is cut in half. What is the new force?

1. One charge triples, so its factor is 3.

2. The distance is cut in half: flip ½ to 2, then square it: $2^2 = 4$, so the distance factor is 4.

3. New force = 0.150 N × 3 × 4 = 1.80 N. A smaller distance gives a stronger force, so a bigger answer is expected.

Slide 2 of 7 · Coulomb’s law

Slide 1 showed how the force changes. One equation holds all of those rules.

Coulomb’s law gives the size of the force between two **point charges**. A point charge is a **tiny dot**. Formula: $F = \mathbf{k|q_1||q_2|/r^2}$, with $k = \mathbf{8.99 \times 10^9\ N{\cdot}m^2/C^2}$. F is in **newtons (N)**.

Bars | | mean **size only**: drop any minus sign. Convert µC to C by multiplying by $\mathbf{10^{-6}}$. The distance r is in **metres**, measured from **centre to centre**.

The signs give the direction: like charges **repel**, unlike charges **attract**. Choose each arrow from the **signs of the charges**, not from a minus sign in the answer.

Newton’s third law: each charge feels the **same size** of force. To find r, rearrange first: multiply both sides by r^2, divide by F, then take the **square root**: $r = \mathbf{\sqrt{(k|q_1||q_2|/F)}}$.

Example. A +3.0 µC sphere and a −5.0 µC sphere are 0.30 m apart. Use $k = 8.99 \times 10^9\ N{\cdot}m^2/C^2$. What is the size of the force on each sphere, and does the force attract or repel?

1. The charges have opposite signs, so the force is attraction.

2. $F = (8.99 \times 10^9\ N{\cdot}m^2/C^2)(3.0 \times 10^{-6}\ C)(5.0 \times 10^{-6}\ C) \div (0.30\ m)^2 = 1.50\ N$.

3. Each sphere feels 1.50 N. Swapping the sphere positions changes only the arrow, never the size.

Try it. A +2.0 µC sphere and a +4.0 µC sphere repel each other with a force of 0.500 N. What is the distance between them?

1. Rearrange: $r^2 = k|q_1||q_2|/F$, so $r = \sqrt{(k|q_1||q_2|/F)}$.

2. Convert first: $2.0\ \mu C = 2.0 \times 10^{-6}\ C$ and $4.0\ \mu C = 4.0 \times 10^{-6}\ C$.

3. $r = \sqrt{[(8.99 \times 10^9)(2.0 \times 10^{-6})(4.0 \times 10^{-6}) \div 0.500]} = 0.379\ m$. Take the square root at the end, or you get r^2 instead of r.

Slide 3 of 7 · Method: what happens to the force

These are the steps you used on slide 1.

Step 1: write the formula **$F = k|q_1||q_2|/r^2$**. The constant k never changes, so it **drops out**.

Step 2: list the **changes** in words, such as “q_1 doubles” or “r triples”.

Step 3: make one **factor** for each change. A charge that changes by n gives a factor of **n**. A distance that changes by n gives **$1/n^2$**: flip it, then square it.

Step 4: **multiply** the factors. Keep the total as a **fraction**, such as $2 \times 2 \times 1/9 = 4/9$.
Step 5: new force = old force × the **total factor**. Check: a bigger charge or a smaller distance must make the force **bigger**.

Example. Two charges exert a force of 0.800 N on each other. Both charges are doubled in size and the distance between them is tripled. What is the new force?

1. Steps 1 and 2: $F = k|q_1||q_2|/r^2$. q_1 doubles, q_2 doubles, r triples.

2. Step 3: the charges give 2 and 2. The distance: flip 3 to 1/3, then square it to get 1/9.

3. Steps 4 and 5: $2 \times 2 \times 1/9 = 4/9$, so the new force is $0.800\ \text{N} \times 4/9 = 0.356\ \text{N}$. The total factor is less than 1, so the force is weaker.

Try it. Two charges exert a force of 0.500 N on each other. The distance is doubled and one charge is tripled. What is the new force? Write each step number beside your work.

1. The distance doubles: flip 2 to 1/2, then square it to get 1/4. One charge triples: factor 3.

2. Total factor = $3 \times 1/4 = 3/4$.

3. New force = $0.500\ \text{N} \times 3/4 = 0.375\ \text{N}$. The total factor is less than 1, so the force is weaker. This matches a longer distance winning over a bigger charge.

Slide 4 of 7 · Method: force between two charges

These are the steps you used on slide 2.

Step 1: list the givens in **coulombs** (μC × 10^{-6}) and **metres**.

Step 2: use the **sizes** of the charges. Drop any minus sign, because the size of a force is never **negative**.

Step 3: put the numbers into $F = \mathbf{k|q_1||q_2|/r^2}$, with $k = \mathbf{8.99 \times 10^9\ N{\cdot}m^2/C^2}$. Square the **distance**.

Step 4: use the signs for the direction. Like charges **repel**; unlike charges **attract**. Each charge feels the **same size** of force. Check: the unit is **newtons** and the power of ten fits the picture.

Example. A +3.0 μC sphere and a −5.0 μC sphere are 0.30 m apart. Use $k = 8.99 \times 10^9$ N·m²/C². What is the size of the force on each sphere, and does the force attract or repel?

1. Step 1: 3.0×10^{-6} C, 5.0×10^{-6} C and 0.30 m.

2. Steps 2 and 3: $F = (8.99 \times 10^9)(3.0 \times 10^{-6})(5.0 \times 10^{-6}) \div (0.30)^2 = 1.498\ldots$ N, which is 1.50 N.

3. Step 4: the signs are opposite, so the force is attraction, 1.50 N on each sphere. Check: newtons, and near 1 N for microcoulombs 0.30 m apart.

Try it. Two +4.0 μC spheres are 0.20 m apart. Use $k = 8.99 \times 10^9$ N·m²/C². What is the size of the force on each sphere, and does the force attract or repel? Write each step number beside your work.

1. Step 1: 4.0×10^{-6} C for each charge and 0.20 m.

2. Steps 2 and 3: $F = (8.99 \times 10^9)(4.0 \times 10^{-6})(4.0 \times 10^{-6}) \div (0.20)^2 = 3.596$ N, which is 3.60 N.

3. Step 4: both charges are positive, so the force is repulsion, 3.60 N on each sphere.

Slide 5 of 7 · How was Coulomb’s law found?

Slide 2 gave Coulomb’s law. How did Coulomb find it, long before modern meters?

In **1785** Coulomb measured the force between two small charged balls with a **torsion balance**. A thin **fibre** holds a light rod. A force on the rod **twists** the fibre.

The twist angle is **proportional** to the force. A scale around the rod shows the **angle**. A thin fibre twists even for a very **small** force.

Coulomb could not measure charge directly. He touched a charged sphere to an **identical** uncharged sphere. The charge is shared **equally**, so each has **q ÷ 2**.

Cavendish used the same kind of twisting balance to measure the **gravitational** force between heavy **lead** masses. Both balances turn a tiny force into a twist **angle**.

Example. A metal sphere carries +8.0 µC. It touches an identical metal sphere that is uncharged. They are then separated. What is the charge on each sphere?

1. Total charge = +8.0 µC + 0 = +8.0 µC.

2. The spheres are identical, so the charge is shared equally. Each sphere gets +8.0 ÷ 2 = +4.0 µC.

Try it. NEW, for teacher review. Say what the scale reading of a torsion balance tells you, and why Coulomb used a very thin fibre.

1. The twist angle is proportional to the force, so a bigger angle means a bigger force.

2. A thin fibre twists a measurable amount even for the very small force between two charged balls.

Slide 6 of 7 · Electric force and gravity

In Physics 20, gravity also had a law with $1/r^2$. Compare it with Coulomb's law from slide 2.

Newton's law of gravitation: $F_g = \mathbf{Gm_1m_2/r^2}$, where $G = \mathbf{6.67 \times 10^{-11}\ N{\cdot}m^2/kg^2}$ and m is mass in **kg**. Coulomb's law: $F_e = \mathbf{k|q_1||q_2|/r^2}$. The two laws have the **same form**.

Both forces depend on a **product** and fall as $\mathbf{1/r^2}$. Both come in **equal and opposite** pairs.

Gravity only **attracts**: there is one kind of mass. The electric force can **attract or repel**: there are two kinds of charge. For an electron and a proton the electric force is about $\mathbf{10^{39}}$ times bigger.

Most large objects are **neutral**: their electric forces cancel, so **gravity** wins. A ratio is one force **divided by** the other. When both use the same r, the $\mathbf{r^2}$ cancels.

Example. An electron and a proton are the same distance r apart. Find the ratio $F_e \div F_g$. Use $k = 8.99 \times 10^9$, $G = 6.67 \times 10^{-11}$, $e = 1.60 \times 10^{-19}$ C, m(proton) $= 1.67 \times 10^{-27}$ kg and m(electron) $= 9.11 \times 10^{-31}$ kg.

1. $F_e = ke^2/r^2$ and $F_g = G\ m_p\ m_e/r^2$. Divide: the distance cancels, so the ratio is the same at any distance.

2. $F_e \div F_g = ke^2 \div (G\ m_p\ m_e)$.

3. $= (8.99 \times 10^9)(1.60 \times 10^{-19})^2 \div [(6.67 \times 10^{-11})(1.67 \times 10^{-27})(9.11 \times 10^{-31})] = 2.27 \times 10^{39}$.

Try it. Check the exponent: in the ratio above, which constants are in the top and which in the bottom? Write the ratio as a formula, then say why the answer is so large.

1. Top: k and e^2 (electric). Bottom: G, m_p and m_e (gravity).

2. The ratio is $ke^2 \div (G\ m_p\ m_e) = 2.27 \times 10^{39}$.

3. It is huge because G and the masses are tiny, so gravity between particles is extremely weak. Check that the exponent is +39, not −39.

Slide 7 of 7 · Two-step problems

B1 shared charge between spheres that touch. Slide 2 found the force. This slide joins the two.

Two events, two steps. The force uses the charges **after touching**, not the ones the spheres started with. So the **charge** comes first and the **force** second.

Step one: share the charge. Identical spheres share the total **equally**. **Add** the signed charges, then divide by **two**. A negative value takes away from a positive one.

Step two: use the new charges in $F = \mathbf{k|q_1||q_2|/r^2}$. Convert µC to **C** and use **metres**. Both charges the same sign give **repulsion**; one of each sign gives **attraction**.

Factor questions: decide which way each change pushes the force. Half the distance: factor **more than 1**. Half one charge: factor **less than 1**. Write both factors, **multiply** them, then multiply by the old force.

Example. Two identical metal spheres carry +6.0 µC and −2.0 µC. They touch and are separated, and then they are placed 0.10 m apart. What is the size of the force on each sphere, and does it attract or repel?

1. After touching, each sphere has $(+6.0 + (-2.0)) \div 2 = +2.0$ µC.

2. $F = (8.99 \times 10^{9})(2.0 \times 10^{-6})(2.0 \times 10^{-6}) \div (0.10)^2 = 3.60$ N.

3. Both charges are positive, so the force is repulsion.

Try it. Two small charged spheres 0.40 m apart exert a force of 2.4 N on each other. They are moved to 0.20 m apart, and the charge on one of them is replaced by half its original size. What is the new force? A. 1.2 N. B. 2.4 N. C. 4.8 N. D. 9.6 N.

1. Halving the distance multiplies the force by 4. Halving one charge multiplies it by 1/2.

2. New force = 2.4 N × 4 × 1/2 = 4.8 N.

3. Answer C.

B2 · Worked practice

Coulomb’s law | Questions 1-2 of 12

Use $k = 8.99 \times 10^9$ N·m²/C² and $g = 9.81$ m/s² where needed. These values are from the Alberta Physics 30 data sheets, Constants section. All point-charge calculations approximate air as vacuum.

Easy · B2-P01. Convert each value to SI units: (a) +4.50 µC, (b) −8.00 nC, (c) 25.0 cm. Keep the charge signs.

1. Micro means 10^{-6}, so +4.50 µC = $+4.50 \times 10^{-6}$ C. Nano means 10^{-9}, so −8.00 nC = -8.00×10^{-9} C.

2. There are 100 cm in 1 m, so 25.0 cm = 0.250 m. Convert before using Coulomb’s law because k is expressed in SI units.

Easy · B2-P02. A positive charge A is left of a negative charge B. Draw the electric-force arrow on each. If the force magnitude on B is 0.240 N, what is the force on A?

1. Unlike charges attract. Draw the arrow on B left, toward A; draw the arrow on A right, toward B.

2. The pair forces have equal magnitudes and opposite directions, so the force on A is 0.240 N right. They act on different objects and do not cancel on one object.

B2 · Worked practice

Easy · B2-P03. Two charges exert 0.720 N. Keep both charges fixed and triple their distance. What is the new force magnitude? Explain the factor.

1. Coulomb’s law gives F proportional to $1/r^2$ when charges are fixed. Tripling r divides the force by $3^2 = 9$.

2. Fnew = 0.720/9 = 0.0800 N. The smaller force is consistent with greater distance.

Easy · B2-P04. Keep the distance and q_2 fixed, but double $|q_1|$. What happens to the electric-force magnitude? Would doubling the distance instead give the same change? Explain.

1. The charge product is in the numerator to the first power, so doubling $|q_1|$ doubles the force.

2. Doubling distance instead makes the denominator four times larger and the force one quarter of its original value. These are different changes.

B2 · Worked practice

Medium · B2-P05. Two point charges in air are A = +3.00 μC on the left and B = +2.00 μC on the right, separated by 0.200 m. What is the force on B, including direction? Use $k = 8.99 \times 10^9$ N·m²/C².

1. Givens from the prompt: $q_A = +3.00 \times 10^{-6}$ C, $q_B = +2.00 \times 10^{-6}$ C, r = 0.200 m. k is the stated data-booklet constant. Like charges repel, so B feels a force right.

2. Use the point-charge model: $F = k|q_A q_B|/r^2 = (8.99 \times 10^9)(3.00 \times 10^{-6})(2.00 \times 10^{-6})/(0.200)^2 = 1.3485$ N.

3. To three significant figures: 1.35 N right on B. The units reduce to N and the direction is away from A.

Medium · B2-P06. Two point charges in air are A = −5.00 nC on the left and B = +8.00 nC on the right, 4.00 cm apart. What is the force on B? Use $k = 8.99 \times 10^9$ N·m²/C².

1. Givens: $|q_A| = 5.00 \times 10^{-9}$ C, $|q_B| = 8.00 \times 10^{-9}$ C, r = 0.0400 m from the prompt; k is given. Unlike signs attract, so B is pulled left.

2. $F = k|q_A q_B|/r^2 = (8.99 \times 10^9)(5.00 \times 10^{-9})(8.00 \times 10^{-9})/(0.0400)^2 = 2.2475 \times 10^{-4}$ N.

3. Answer: 2.25×10^{-4} N left. The minus sign on q_A identifies attraction; it does not make a force magnitude negative.

B2 · Worked practice

Medium · B2-P07. Two point charges have magnitudes 2.00 µC and 5.00 µC. Their force magnitude in air is 0.500 N. How far apart are they? Use $k = 8.99 \times 10^9$ N·m²/C². Can the force direction be determined from these data?

1. Givens: $|q_1| = 2.00 \times 10^{-6}$ C, $|q_2| = 5.00 \times 10^{-6}$ C and F = 0.500 N from the prompt. Use Coulomb’s law because it relates these charges, force and distance.

2. Rearrange before substitution: $Fr^2 = k|q_1||q_2|$, then $r = \sqrt{(k|q_1||q_2|/F)} = \sqrt{(0.0899/0.500)} = 0.424$ m.

3. A distance is positive. Signs and positions were not given, so neither attraction/repulsion nor an absolute force direction is determined.

Medium · B2-P08. With the charges fixed, doubling the distance changes an electric-force magnitude from 0.800 N to what value? Apply the same change to a gravitational force initially 0.120 N. Explain one similarity and one difference between the two forces.

1. Both laws have r^2 in the denominator. Doubling distance multiplies the denominator by four: electric force = 0.800/4 = 0.200 N; gravitational force = 0.120/4 = 0.0300 N.

2. Both are inverse-square relationships for the point-source models. Gravity between ordinary masses attracts; electric force may attract or repel depending on the charge signs. The same $1/r^2$ pattern does not mean the same strength.

B2 · Worked practice

Challenging · B2-P09. The initial force magnitude is 0.540 N. Then $|q_1|$ is doubled, $|q_2|$ is tripled, and distance is tripled. The signs stay the same. What is the new magnitude, and does the force change between attraction and repulsion?

1. The numerator changes by $2 \times 3 = 6$. The squared distance changes by $3^2 = 9$. Thus $F_{new}/F_{old} = 6/9 = 2/3$.

2. $F_{new} = 0.540 \times (2/3) = 0.360$ N. Combining the factors before calculating avoids confusing a charge factor with a distance-square factor.

3. The signs remain unchanged, so it is still the same kind of force. An absolute direction would also require the positions and the chosen object.

Challenging · B2-P10. Sample data for fixed charges are: at r = 0.100 m, F = 0.800 N; at 0.200 m, F = 0.200 N; at 0.400 m, F = 0.0500 N. Draw a straightened graph with labelled axes and units, plot all three points, find its slope, and predict F at 0.300 m.

1. An inverse-square model becomes linear when the horizontal variable is $1/r^2$. Plot x = 100, 25.0 and 6.25 m^{-2} against y = 0.800, 0.200 and 0.0500 N. Use numerical scales and draw the line through the origin.

2. Slope = $\Delta F/\Delta(1/r^2) = (0.800 - 0.200)/(100 - 25.0) = 0.00800$ N·m^2. The third point also fits this slope.

3. At r = 0.300 m, $1/r^2 = 11.111\ldots$ m^{-2}, so $F = (0.00800)(11.111\ldots) = 0.0889$ N. These are sample data, not independent experimental proof of the law.

B2 · Worked practice

Coulomb’s law | Questions 11-12 of 12

Challenging · B2-P11. A student uses $q_1 = +2.00\ \mu C$, $q_2 = -6.00\ \mu C$ and $r = 0.300$ m, then reports “$F = -1.20$ N, so the force on the right-hand charge points left.” Say which part is right and which part is wrong. What is the correct magnitude and force direction on each charge? Use $k = 8.99 \times 10^9\ N\cdot m^2/C^2$; q_1 is on the left.

1. Coulomb’s magnitude law uses $|q_1||q_2|$. $F = (8.99 \times 10^9)(2.00 \times 10^{-6})(6.00 \times 10^{-6})/(0.300)^2 = 1.1987\ N = 1.20\ N$.

2. Unlike signs attract: the right-hand charge q_2 feels 1.20 N left; the left-hand charge q_1 feels 1.20 N right. These conclusions require identifying the object and positions.

3. A negative charge product indicates unlike signs, not a universal left direction. If right is chosen positive, the signed x-component on q_2 is -1.20 N, while its magnitude remains $+1.20$ N.

Challenging · B2-P12. You can vary distance between two small charged spheres and measure force. What should remain fixed to test the distance relationship? Choose at least three distances, predict the force ratios for r, 2r and 3r, name a suitable graph, and explain one role shared by the Coulomb and Cavendish torsion balances.

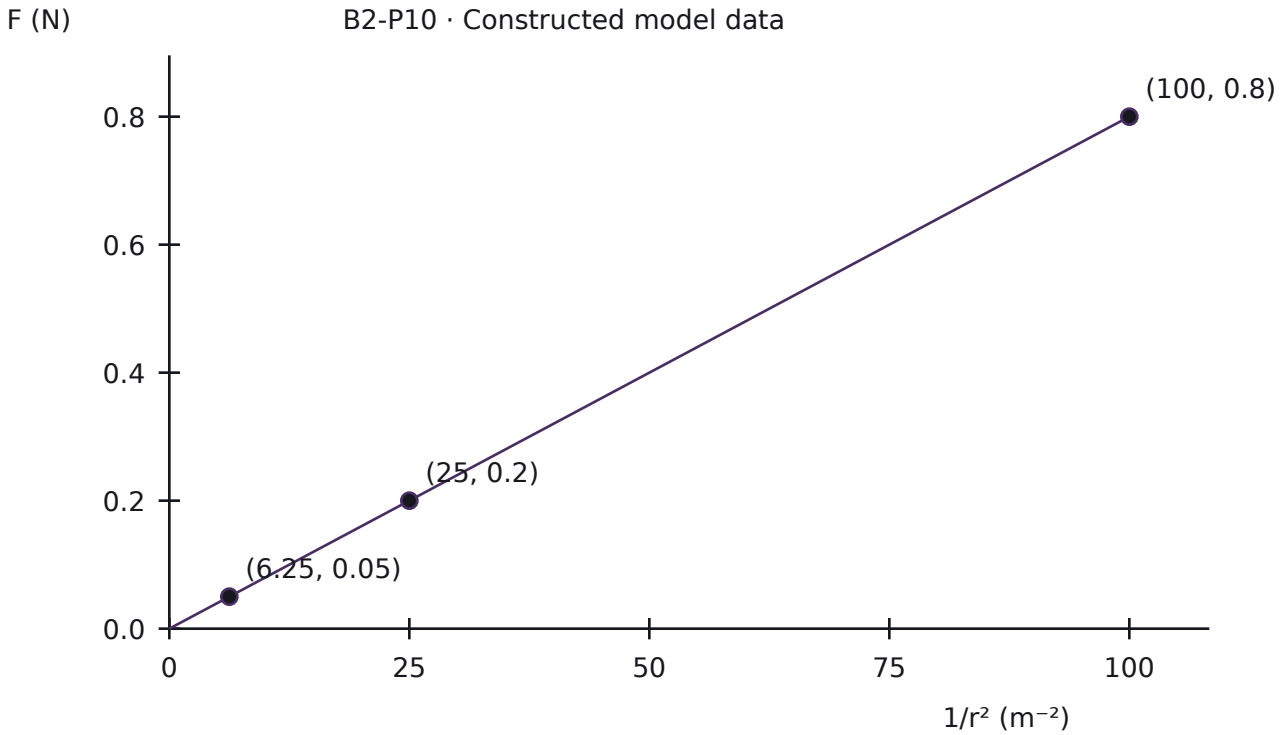
1. Keep both charges and the surrounding medium fixed; use distances large compared with sphere size for the point-charge approximation. A possible set is 0.100, 0.200 and 0.300 m.

2. Predicted ratios are 1 : 1/4 : 1/9. Plot force in N against $1/r^2$ in m^{-2}; an ideal inverse-square model gives a straight line through the origin.

3. Both balances use the twist of a suspended wire to make a small force measurable. Coulomb studied electric forces; Cavendish studied gravitational attraction. Charge leaking away or inaccurate centre-to-centre distances could distort the test.

B2-P10 · Graph key

Straightened force-distance data | all three points plotted



Slope = (0.800 - 0.200)/(100 - 25.0) = 0.00800 N·m². A horizontal value of 11.111... m⁻² corresponds to r = 0.300 m, giving F = 0.0889 N.